SOLUTIONS MANUAL FOR SELECTED SOLUTIONS MANUAL FOR SELECTED

PROBLEMS IN PROBLEMS IN

PROCESS SYSTEMS ANALYSIS AND CONTROL

DONALD R. COUGHANOWR
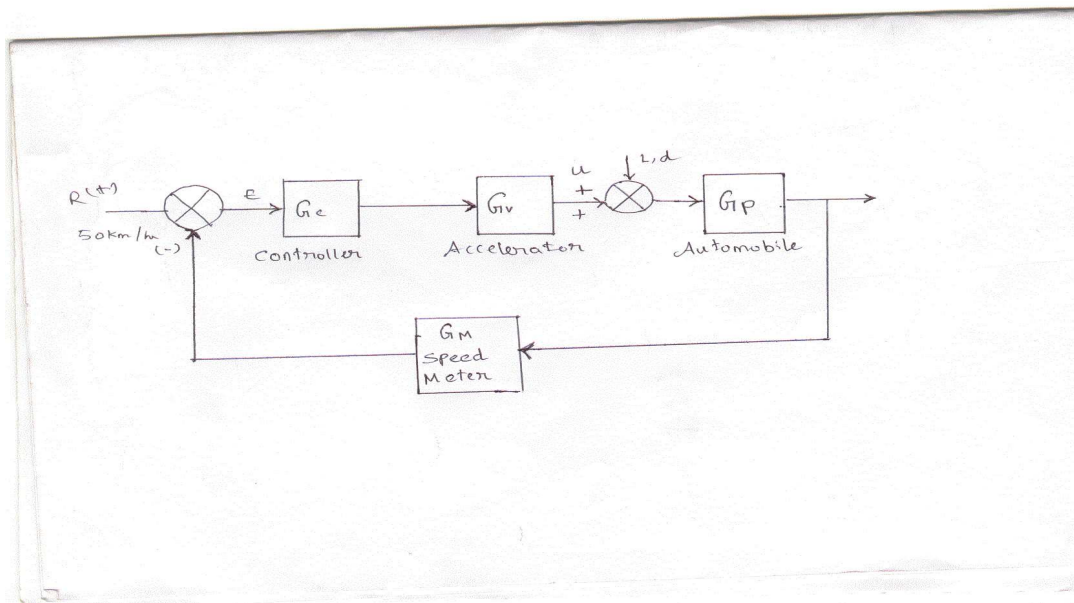
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PART 1:   SOLUTIONS FOR SELECTED PROBLEMS
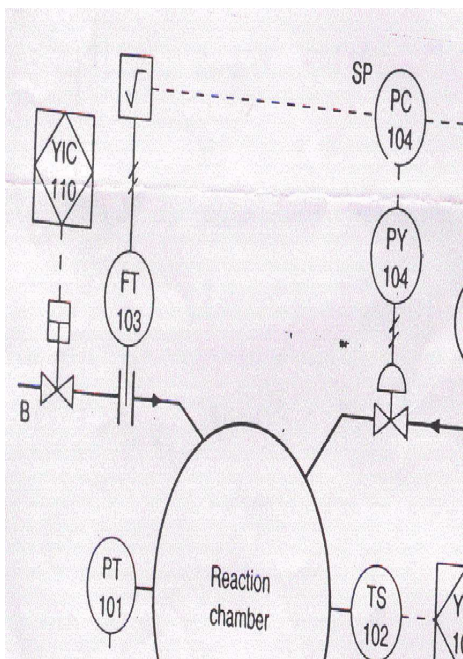
PART2:   LIST OF USEFUL BOOKS

PART3:   USEFUL WEBSITES

PART 1

1.1 Draw a block diagram for the control system generated when a human being steers an automobile.
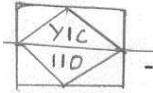

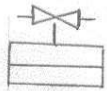**1.2 From the given figure specify the devices**
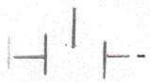
Solution:

Loop 110

 - Sequential indicating controller. Y represents mathematical symbols. Emits signal to valve which controls the flow rate of B.
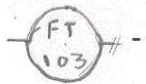
 - solenoidal valve. It operates on electric signal.

Loop 103

 - Orifice meter, Measures flow rate.

 - Flow transmitter. It converts flow to pneumatic signal of 20 to 100 Mpa.
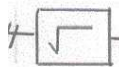
 - Removes the square root from signal and makes it linear. Input is pneumatic and output is electrical.

Loop 104
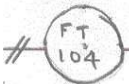
 - Flow transmitter converts flow of A to a pneumatic signal of 20-100 Kpa.
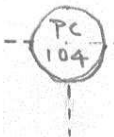
 - Pressure controller. It receives two electrical signals one from flow

- Transmitter of loops 103 and 104.Maintain ratio of A and B in reaction chamber.

 - IP converter, it converts standard electric signal to pneumatic signal.

 - A control valve. Input is a standard pneumatic signal and controls the flow rate

Loop 101

PT
101
— Pressure transmitter, Measures pressure and given electric signal of 4 20 Ma.

PC
101
— Computer . Sends digital data to pressure controller. It is used to set the set point and placed in control room.

PC
101
— Pressures controller - has two inputs one digital signal from computer and one electrical signal from pressure transmitter.

M
101
— Motor controlled valve, controls the flow rate of C in the chamber.

Loop 102

TS
102
— Temperature safety.

YIC
102
— Sequential indicating controller.

Loop 112

YIC
112
— Sequential indicating controller
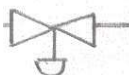
— Solenoidal valve. Controls flow rate of vent gases

Inversion by partial   fractions:

3.1(a) $\dfrac{dx^2}{dt^2} + \dfrac{dx}{dt} + x = 1 \quad x(0) = x^{'}(0) = 0$

$$L\left[\dfrac{dx^2}{dt^2}\right] = s^2 X(s) - sx(0) - x^{'}(0)$$

$$L\left[\dfrac{dx}{dt}\right] = s\ X(s) - x(0)$$

L(x) = X(s)

$L\{1\} = 1/s$

$$s^2 X(s) - sx(0) - x'(0) + s\ X(s) - x(0) + X(s) = \frac{1}{s}$$

$$= (s^2 + s + 1)X(s) = \frac{1}{s}$$

$$X(s) = \frac{1}{s(s^2 + s + 1)}$$

Now, applying partial fractions splitting, we get

$$X(s) = \frac{1}{s} - \frac{s+1}{(s^2 + s + 1)}$$

$$X(s) = \frac{1}{s} - \frac{s+1}{\left(s + \frac{1}{2}\right)^2 + \left[\frac{\sqrt{3}}{2}\right]^2} - \left(\frac{1}{2}\right)\left(\frac{2}{\sqrt{3}}\right)\frac{\frac{\sqrt{3}}{2}}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$L^{-1}(X(s)) = 1 - e^{-\frac{1}{2}t} Cos\frac{\sqrt{3}}{2}t - \frac{1}{\sqrt{3}} e^{-\frac{1}{2}t} \sin\frac{\sqrt{3}}{2}t$$

$$X(t) = 1 - e^{-\frac{1}{2}t}\left(Cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}} Sin\left(\frac{\sqrt{3}}{2}t\right)\right)$$

b) $\dfrac{dx^2}{dt^2} + 2\dfrac{dx}{dt} + x = 1\ \ x(0) = x'(0) = 0$

when the initial conditions are zero, the transformed equation is

$$(s^2 + s + 1)X(s) = \frac{1}{s}$$

$$X(s) = \frac{1}{s(s^2 + s + 1)}$$

$$\frac{1}{s(s^2 + s + 1)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2s + 1}$$

$$1 = A(s^2 + 2s + 1) + Bs^2 + Cs$$

$0 = A + B (by\ equating\ the\ co-effecient\ of\ s^2)$

$0 = 2A + C (by\ equating\ the\ co-effecients\ of\ s)$

$1 = A (by\ equating\ the\ co-effecients\ of\ const)$

$A + B = 0$

$B = -1$

$C = -2A$

$A = 1, B = -1, C = -2$

$$X(s) = \frac{1}{s} - \frac{s + 2}{s^2 + 2s + 1}$$

$$L^{-1}\{X(s)\} = L^{-1}\left\{\frac{1}{s} - \frac{(s+1)+1}{(s+1)^2}\right\}$$

$$\{X(t)\} = 1 - L^{-1}\left\{\frac{1}{s+1} + \frac{1}{(s+1)^2}\right\}$$

$$\{X(t)\} = 1 - e^{-t}(1 + t)$$

3.1   C $\dfrac{dx^2}{dt^2} + 3\dfrac{dx}{dt} + x = 1$ $x(0) = x'(0) = 0$

by Applying laplace transforms, we get

$$= (s^2 + 3s + 1)X(s) = \frac{1}{s}$$

$$X(s) = \frac{1}{s(s^2 + 3s + 1)}$$

$$X(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 3s + 1}$$

$$1 = A(s^2 + 3s + 1) + Bs^2 + Cs$$
$$0 = A + B(by \; equating \; the \; co-effecient \; of \; s^2)$$
$$0 = 3A + C(by \; equating \; the \; co-effecients \; of \; s)$$

$$1 = A(by \; equating \; the \; co-effecients \; of \; const)$$
$$A + B = 0$$
$$B = -1$$
$$C = -3A = -3$$
$$A = 1, B = -1, C = -3$$

$$L^{-1}\{X(s)\} = L^{-1}\left\{\frac{1}{s} - \frac{s+3}{s^2 + 3s + 1}\right\}$$

$$L^{-1}\{X(s)\} = L^{-1}\left\{\frac{1}{s} - \frac{s+3}{\left(s+\frac{3}{2}\right)^2 - \left(\frac{\sqrt{5}}{2}\right)^2}\right\}$$

$$L^{-1}\{X(s)\} = L^{-1}\left\{\frac{1}{s} - \frac{s+\frac{3}{2}}{\left(s+\frac{3}{2}\right)^2 - \left(\frac{\sqrt{5}}{2}\right)^2} - \frac{\left(\frac{3}{2} \cdot \frac{2}{\sqrt{5}}\right)\frac{\sqrt{5}}{2}}{\left(s+\frac{3}{2}\right)^2 - \left(\frac{\sqrt{5}}{2}\right)^2}\right\}$$

$$X(t) = 1 - e^{-\frac{3t}{2}}\left(Cos\frac{\sqrt{5}t}{2} + \frac{3}{\sqrt{5}}\sinh\frac{\sqrt{5}}{2}t\right)$$

3.2(a)
$$\frac{dx^4}{dt^4} + \frac{d^3x}{dt^3} = Cos \; t; x(0) = x'(0) = x''(0) = 0$$
$$x^{11}(0) = 1$$

Applying Laplace transforms, we get

$$s^4 X(s) - s^3 x(0) - s^2 x^1(0) - s x''(0) - x'''(0) + s^3 X(s) - s^2 x(0) - s x'(0) - x''(0) = \frac{s}{s^2+1}$$

$$X(s)(s^4+s^3) - (s+1) = \frac{s}{s^2+1}$$

$$X(s) = \left((\frac{s}{s^2+1} + 1) + (s+1) \right) \bigg/ s^4 + s^3$$

$$= \frac{s+s^3+s+s^2+1}{s^3(s^2+1)(s+1)} = \frac{s^3+s^2+2s+1}{s^3(s^2+1)(s+1)}$$

$$\frac{s^3+s^2+2s+1}{s^3(s^2+1)(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s+1} + \frac{Es+F}{s^2+1}$$

$$s^3+s^2+2s+1 = As^2(s+1)(s^2+1) + Bs(s+1)(s^2+1) + c(s+1)(s^2+1) + Ds^3(s^2+1) + (Es+F)s^3(s+1)$$

A+B+E=0 equating the co-efficient of s^5.
A+B+E+F=0 equating the co-efficient of s^4.
A+B+C+D+F=0 equating the co-efficient of s^3.
A+B+C=0 equating the co-efficient of s^2.
B+C=2 equating the co-efficient of s.
A+B+E=0 equating the co-efficient of s^2.
C=1equating the co-efficient constant.

C=1
-B=-C+2=1
A=1-B-C=-1
D+F=0
E+F=0D+E=1
D-E=0
2D=1

A=-1; B=1; C=1
D=1/2; E=1/2; F =-1/2

$$L^{-1}\{(s)\} = L^{-1}\left\{\frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s^3} + \frac{1/2}{s+1} + \frac{1/2(s-1)}{s^2+1}\right\}$$

$$L^{-1}\{X(s)\} = L^{-1}\left\{\frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s^3} + \frac{1/2}{s+1} + \frac{1/2(s-1)}{s^2+1}\right\}$$

$$\{X(t)\} = -1 + t + \frac{t^2}{2} + \frac{1}{2}e^{-t} + \frac{1}{2}Cos\,t - \frac{1}{2}S\,\text{int}$$

$$\frac{d^2q}{dt^2} + \frac{dq}{dt} = t^2 + 2t \quad q(0) = 4; q^1(0) = -2$$

applying laplace transforms,we get

$$s^2Q(s) - sq(0) - q^{'}(0) + sQ((s) - q(0) = \frac{2}{s^3} + \frac{2}{s^2}$$

$$Q(s)(s^2 + s) - 4s + 2 - 4 = \frac{2}{s^2}\left(\frac{1}{s} + 1\right)$$

$$Q(s) = \frac{\dfrac{2(s+1)}{s^3} + (4s+2)}{(s^2+s)}$$

$$= \frac{2s + 2 + 4s^4 + 2s^3}{s^4(s+1)}$$

$$Q(s) = 4\left(\frac{1}{s+1}\right) + \frac{2}{s(s+1)} + \frac{2*3}{s^4(s+1)}$$

$$L^{-1}(Q(s)) = q(t) = 4e^{-t} + 2(1 - e^{-t}) + \frac{1}{3}t^3$$

therefore $\quad q(t) = 2 + \dfrac{t^3}{3} + 2e^{-t}$

3.3 a) $\dfrac{3s}{(s^2+1)(s^2+4)} = \dfrac{3s}{3}\left[\dfrac{1}{s^2+1} - \dfrac{1}{s^2+4}\right]$

$= \left[\dfrac{1}{s^2+1^2} - \dfrac{1}{s^2+2^2}\right]$

$L^{-1}\left[\dfrac{1}{s^2+1^2} - \dfrac{1}{s^2+2^2}\right] = Cost - Cos2t$

b) $\dfrac{1}{s(s^2-2s+5)} = \dfrac{1}{s\left[(s-1)^2+2^2\right]} = \dfrac{A}{s} + \dfrac{B+C}{s^2-2s+5}$

A+B=0
-2A+C=0
5A=1
A=1/5 ;B=-1/5;C=2/5

We get $\qquad X(s) = \dfrac{1}{5}\left[\dfrac{1}{s} + \dfrac{2-s}{s^2-2s+5}\right]$

Inverting,we get

$= \dfrac{1}{5}\left[1 + \dfrac{1}{2}e^t Sin2t - e^t Cos2t\right]$

$= \dfrac{1}{5}\left[1 + e^t\left(\dfrac{1}{2}Sin2t - Cos2t\right)\right]$

c) $\dfrac{3s^2-s^2-3s+2}{s^2(s-1)^2} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{s-1} + \dfrac{D}{(s-1)^2}$

$$As(s-1)^2 + B(s-1)^2 + Cs^2(s-1) + Ds^2 = 3s^3 - s^2 - 3s + 2$$

$$A(s^3 - 2s + s) + B(s^2 - 2s + 1) + C(s^3 - s^2) + Ds^2 = 3s^3 - s^2 - 3s + 2$$

A+C=3
-2A+B-C+D=-1
A-2B=-3
B=2;
A=2(2)-3=1
C=3-1=2
D=2(1)-2+2-1=1

We get $X(s) = \dfrac{1}{s} + \dfrac{2}{s^2} + \dfrac{2}{s+1} + \dfrac{1}{(s-1)^2}$

By inverse L.T

$$L^{-1}[X(t)] = 1 + 2t + 2e^t + te^t$$
$$L^{-1}[X(t)] = 1 + 2t + e^t(2+t)$$

3.4 Expand the following function by partial fraction expansion. Do not evaluate co-efficient or invert expressions

$$X(s) = \frac{2}{(s+1)(s^2+1)^2(s+3)}$$

$$X(s) = \frac{A}{s+1} + \frac{Bs+C}{s^2+1} + \frac{Ds+E}{(s^2+1)^2} + \frac{F}{s+3}$$

$$= A(s^2+1)^2(s+3) + (Bs+C)(s+1)(s+3)(s^2+1) + (Ds+E)(s+1)(s+3) + F(s+1)(s^2+1)^2$$

$$= A(s^4+2s^2+1)(s+3) + (Bs+C)(s^2+4s+3)(s^2+1) + (Ds+E)(s^2+4s+3) + F(s+1)(s^2+4s+1)$$

$$= s^5(A+B+F) + s^4(3A+C+4B+F) + s^3(2A+B+4C+3B) + s^2(6A+C+4B+3C) + s(A+4C+3B$$
$$+4E+F) + 3A+3AC+3E+F = 2$$

A+B+F=0

-3A+C+4B+F=0

2A+B+4C+3B=0

6A+C+4B+3C=0

A+4C+3B+3D+4E+F=0

3A+3C+3E+F=2

by solving above 6 equations, we can get the values of A,B,C,D,E and

$$X(s) = \frac{1}{s^3(s+1)(s+1)(s+3)^3}.$$

$$X(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s+1} + \frac{E}{s+2} + \frac{F}{s+3} + \frac{G}{(s+3)^2} + \frac{H}{(s+3)^3}$$

by comparing powers of s we can evaluate A,B,C,D,E,F,G and H.

c) $X(s) = \dfrac{1}{s(s+2)(s+3)(s+4)}$

$$X(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3} + \frac{D}{s+4}$$

by comparing powers of s we can evaluate A,B,C,D

3.5 a) $X(s) = \dfrac{1}{s(s+1)(0.5s+1)}$

$Let \ \dfrac{1}{s(s+1)(0.5s+1)} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{(0.5s+1)}$

$$= A\left(\frac{s^2}{2} + \frac{3s}{2} + 1\right) + B\left(\frac{s^2}{2} + s\right) + C(s^2 + s) = 1$$

A=1

$\dfrac{A}{2} + \dfrac{B}{2} + C = 0 = \dfrac{B}{2} + C = -\dfrac{1}{2}$

$\dfrac{3A}{2} + B + C = 0 = B + C = -\dfrac{3}{2}$

B/2=1/2 *-3/2=-1;

B=-2;
C= -3/2+2=1/2

$$X(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{1}{2}\left(\frac{1}{0.5s+1}\right)$$

$$= L^{-1}(X(s)) = x(t) = 1 - 2e^{-t} + e^{-2t}$$

b) $\dfrac{dx}{dt} + 2x = 2;\ x(0) = 0$

Applying laplace trafsorms

$$sX(s) - x(0) + 2X(s) = 2/s$$

$$L^{-1}(X(s)) = \frac{2}{s(s+2)}$$

$$L^{-1}(X(s)) = 2L^{-1}\left[\frac{2}{s(s+2)}\right]$$

$$= L^{-1}(X(s)) = 2L^{-1}\left[\frac{1/2}{s} - \frac{1/2}{s+2}\right]$$

$$= 1 - e^{-2t}$$

3.6 a) $Y(s) = \dfrac{s+1}{s^2 + 2s + 5}$

$$= Y(s) = \frac{s+1}{s^2 + 2s + 5}$$

$$= \frac{s+1}{(s+1)^2 + 4}$$

$$= L^{-1}(Y(s)) = L^{-1}\left[\frac{s+1}{(s+1)^2 + 4}\right]$$

using the table,we get

$$Y(t) = e^{-t}Cos2t$$

b) $$Y(s) = \frac{s^2 + 2s}{s^4}$$

$$Y(s) = \frac{1}{s^2} + \frac{2}{s^3}$$

Y(t)= $L^{-1}(Y(s)) = t + t^2$

c) $$Y(s) = \frac{2s}{(s-1)^3}$$

$$= \frac{2s - 2 + 2}{(s-1)^3}$$

$$= \frac{2}{(s-1)^2} + \frac{2}{(s-1)^3}$$

$$Y(t) = L^{-1}\left[\frac{2}{(s-1)^2}\right] + L^{-1}\left(\frac{2}{(s-1)^3}\right)$$

$$= 2(te^t + \frac{t^2}{2}e^t = e^t(t^2 + 2t)$$

3.7a) $\quad Y(s) = \dfrac{1}{(s^2+1)} = \dfrac{As+B}{(s^2+1)} + \dfrac{Cs+D}{(s^2+1)}$

thus $(As+B) + (Cs+D)(s^2+1) = 1$

$= Cs^3 + Ds^2 + (A+C)s + (B+D) = 1$

C=0,D=0
Also   A=0;B=1

$Y(s) = \dfrac{1}{(s^2+1)^2} = \dfrac{1}{(s+i)^2(s-i)^2} = \dfrac{A}{(s+i)} + \dfrac{B}{(s+i)^2} + \dfrac{C}{(s-i)} + \dfrac{D}{(s-i)^2}$

$A(s+i)(s-i)^2 + B(s-i)^2 + C(s-i)(s+i)^2 + D(s+i)^2 = 1$

$(A+C)s^3 + (-Ai+B+Ci+D)s^2 + (A-2Bi+C+2Di) + (-Ai-B+Ci-D) = 1$

Thus,A+C=0
-Ai+B+Ci+2Di=0   ; B=D
A-2Bi+C+2Di=0
-Ai-B+Ci-D=1   Also D=-Ci;B=-Ci,
                                 A=-C,C=-i/4

A=i/4 ; B=-1/4; D=-1/4

$Y(s) = \dfrac{i/4}{(s+i)} + \dfrac{-1/4}{(s+i)^2} + \dfrac{-i/4}{(s-i)} + \dfrac{-1/4}{(s-i)^2}$

$Y(t) = \dfrac{i/4}{(s+i)} + \dfrac{-1/4}{(s+i)^2} + \dfrac{-i/4}{(s-i)} + \dfrac{-1/4}{(s-i)^2}$

$Y(t) = \dfrac{i/4}{(s+i)} + \dfrac{-1/4}{(s+i)^2} + \dfrac{-i/4}{(s-i)} + \dfrac{-1/4}{(s-i)^2}$

$$Y(t) = i/4e^{-it} - 1/4e^{-it} - 1/4e^{it} - 1/4te^{it}$$

$$Y(t) = 1/4(ie^{-it} - te^{-it} - ie^{it} - te^{it})$$
$$Y(t) = 1/4(i(Cost - iSint) - t(Cost - i\ Sint) - i(Cost + iSint) - t(Cost + i\ Sint))$$

$$Y(t) = 1/4(2\ Sint - 2t\ Cost)$$

$$Y\ (t)\ =\ 1\ /\ 2\ (\ Sin\quad t\ -\quad t\ Cos\quad t\ )$$

3.8 $f(s) = \dfrac{1}{s^2(s+1)}$

$=\ f(s) = \dfrac{A}{s^2} + \dfrac{B}{s} + \dfrac{C}{s+1}$

$=\ A(s+1) + Bs(s+1) + Cs^2 = 1$

Let s=0 ; A=1
    s=1; 2A+B+C=1
s=-1: C=1
B=-1

$f(s) = \dfrac{1}{s^2} + \dfrac{1}{s} + \dfrac{1}{s+1}$

$$f(t) = (t-1) + e^{-t}$$

PROPERTIES OF TRANSFORMS

4.1 If a forcing function f(t) has the laplace transforms

$f(s) = \dfrac{1}{s} + \dfrac{e^{-s} - e^{-2s}}{s^2} - \dfrac{e^{-3s}}{s}$

$= \dfrac{1 - e^{-3s}}{s} + \dfrac{e^{-s} - e^{-2s}}{s^2}$

$$f(t) = L^{-1}\{f(s)\} = [u(t) - u(t-3)] + [(t-1)u(t-1) - (t-2)u(t-2)]$$

$$= u(t) + (t-1)u(t-1) - (t-2)u(t-2) - u(t-3)$$

graph the function f(t)
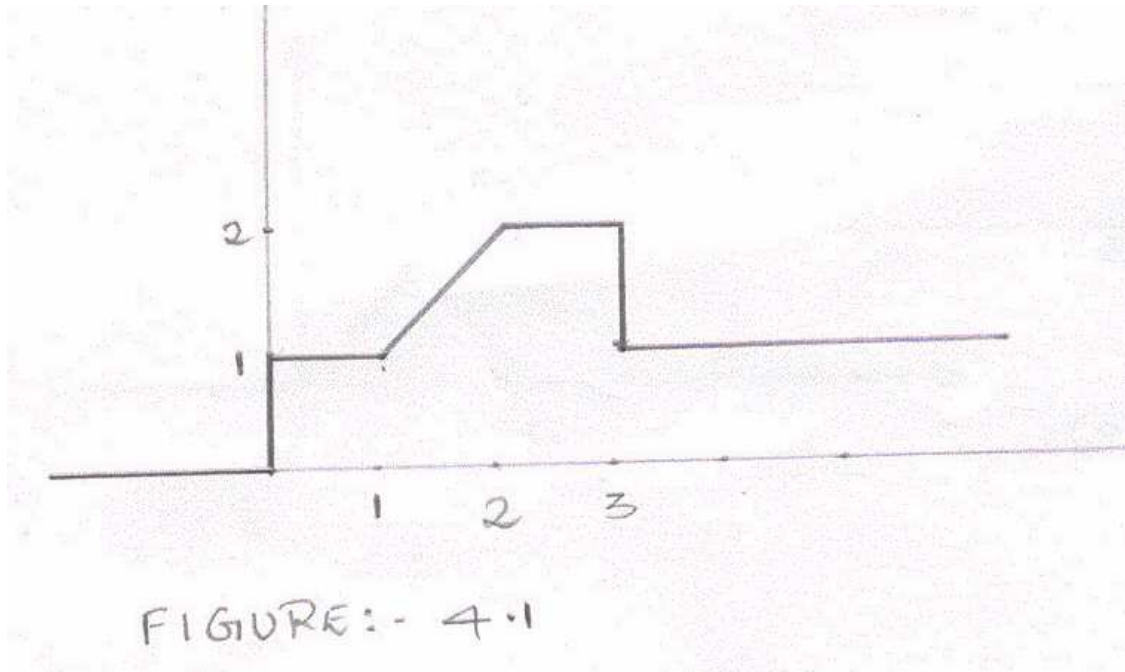
FIGURE:- 4.1

4.2 Solve the following equation for y(t):

$$\int_0^t y(\tau)\,d\tau = \frac{dy(t)}{dt} \quad y(0) = 1$$

Taking Laplace transforms on both sides

$$L\{\int_0^t y(\tau)\,dt\} = L\left\{\frac{dy(t)}{dt}\right\}$$

$$\frac{1}{s}.y(s) = s.y(s) - y(0)$$

$$\frac{1}{s} \cdot y(s) = s \cdot y(s) - 1$$

$$y(s) = \frac{s}{s^2 - 1}$$

$$y(t) = L^{-1}\{y(s)\} = L^{-1}\left\{\frac{s}{s^2 - 1}\right\} \cosh(t)$$

4.3 Express the function given in figure given below the   t – domain and the s-

domain

This graph can be expressed as

$$= \{u(t-1) - u(t-5)\} + \{(t-2)u(t-2) - (t-3)u(t-3)\} + \{u(t-5) - (t-5)u(t-5) + (t-6)u(t-6)\}$$

$$f(t) = u(t-1) + (t-2)u(t-2) - (t-2)u(t-3) - (t-5)u(t-5) + (t-6)u(t-6)$$

$$f(s) = L\{f(t)\} = \frac{e^{-s}}{s} + \frac{e^{-2s}}{s^2} - \frac{e^{-3s}}{s^2} - \frac{e^{-3s}}{s} - \frac{e^{-5s}}{s^2} + \frac{e^{-6s}}{s^2}$$

$$= \frac{e^{-s} - e^{-3s}}{s} + \frac{e^{-2s} + e^{-6s} - e^{-3s} - e^{-5s}}{s^2}$$
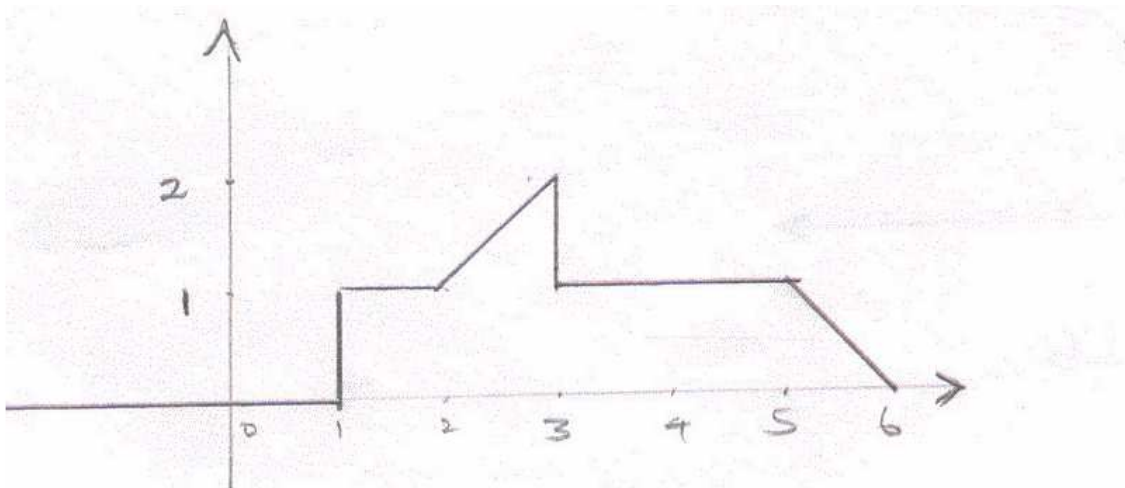
This graph can be expressed as
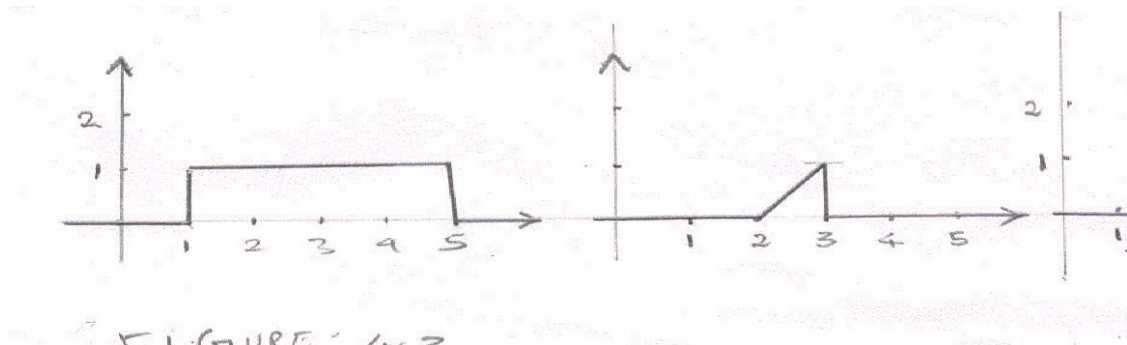
FIGURE: 4.3

4.4 Sketch the following functions:

$$f(t) = u(t) - 2u(t-1) + u(t-3)$$
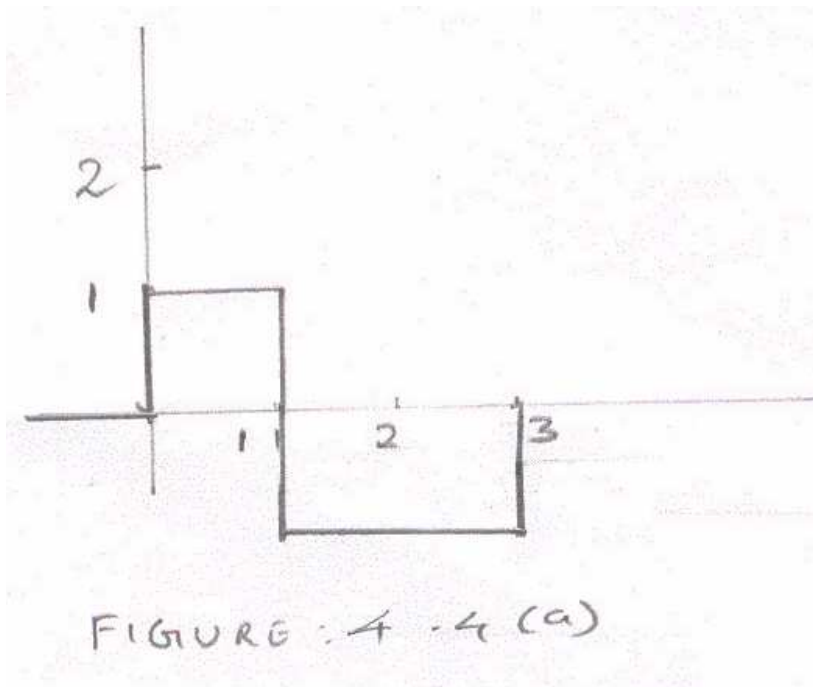
FIGURE: 4.4 (a)

$$f(t) = 3tu(t) - 3u(t-1) + u(t-2)$$

FIGURE 4.4b
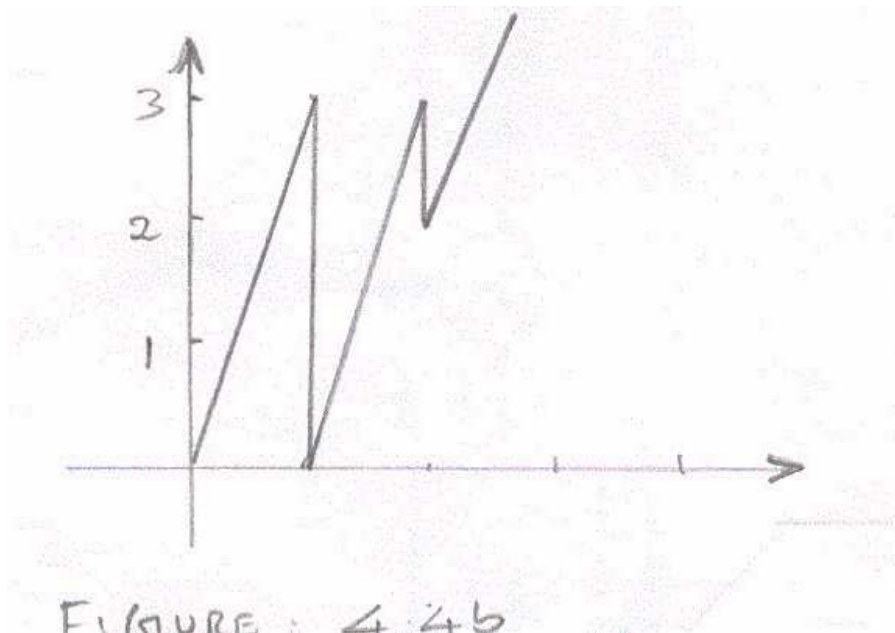
4.5 The function f(t) has the Laplace transform

$$f(S) = (1 - 2e^{-s} + e^{-2s})/s^2$$

obtain the function f(t) and graph f(t)

$$f(s) = \frac{1 - 2e^{-s} + e^{-2s}}{s^2}$$

$$= \frac{1 - e^{-s}}{s^2} - \frac{e^{-s} - e^{-2s}}{s^2}$$

$$f(t) = L^{-1}\{f(s)\} = -(t-1)u(t-1) + tu(t) - \{(t-1)u(t-1) - (t-2)u(t-2)]$$

$$= tu(t) - 2(t-1)u(t-1) + (t-2)u(t-2)$$

4.6 Determine f(t) at t = 1.5 and at t = 3 for following function:

$$f(t) = 0.5u(t) - 0.5u(t-1) + (t-3)u(t-2)$$

At t = 1.5

$$f(t) = 0.5u(t) - 0.5u(t-1) + (t-3)u(t-2)$$

$$f(1.5) = 0.5u(t) - 0.5u(t-1)$$

$$f(1.5) = 0.5 - 0.5 = 0$$

At t = 3

$$f(3) = 0.5 - 0.5 + (3-3) = 0$$

RESPONSE   OF A FIRST ORDER SYSTEMS

5.1 A thermometer having a time constant   of 0.2   min   is placed   in a temperature bath and   after   the thermometer   comes   to equilibrium with the bath, the temperature of the bath is increased linearly with time at the rate of I deg C / min   what is the difference   between   the indicated temperature and bath temperature

 (a) 0.1 min

(b) 10. min

after the change in temperature begins.

© what   is the maximum deviation   between   the indicated temperaturew and bath temperature and when does it occurs.

(d) plot the forcing function and the response on the same graph. After the long enough time buy how many minutes does the response lag the input.

 Consider thermometer to be   in equilibrium   with   temperature   bath at temperature Xs

$$X(t) = X_S + (1°/m)t, t > 0$$

 as it is given that   the temperture varies linearly

X(t)-Xs = t

Let X(t) = X(t) - Xs = t

$Y(s) = G(s).X(s)$

$$Y(s) = \frac{1}{1+\tau s}\frac{1}{s^2} = \frac{A}{1+\tau s} + \frac{B}{s} + \frac{C}{s^2}$$

$A = \tau^2 \quad B = -\tau \quad C = 1$

$$Y(s) = \frac{\tau^2}{1+\tau s} - \frac{\tau}{s} + \frac{1}{s^2}$$

$Y(t) = \tau e^{-t/\tau} - \tau + t$

(a) the difference between the indicated temperature and bath temperature
at t = 0.1 min = X(0.1)_ Y(0.1)

= 0.1 - (0.2e$^{-0.1/0.2}$ - 0.2+0.1) since T = 0.2 given
= 0.0787 deg C

(b) t = 1.0 min

X(1) - Y(1) = 1- (0.2e$^{-1/0.}$2 - 0.2 +1) = 0.1986

(c) Deviation D = -Y(t) +X(t)

$= -\tau e^{-t/T}+T = \tau (-e^{-t/T}+1)$

For maximum value dD/dT = $\tau (-e^{-t/T}+(\_-1/T)) = 0$

-e$^{-t/}$ = 0
 as t tend to infinitive
D = $\tau (-e^{-t/T}+(\_-1/T)) = \tau = 0.2$ deg C

5.2 A mercury thermometer bulb in ½ in . long by 1/8 in diameter. The glass envelope is very thin. Calculate the time constant in water flowing at 10 ft / sec at a temperature of 100 deg F. In your solution , give a summary which includes

(a) Assumptions used.
(b) Source of data
(c) Results

$$T = mCp/hA = \frac{(\rho AL)C_p}{h(A + \pi DL)}$$

Calculation of

$$NU_d = \frac{hD}{K} = CR_e^m \, (\text{Pr})^n$$

$$Re_d = \frac{Dv\rho}{\mu} = \frac{(1/8 * 2.54 * 10^{-2})(10 * 0.3048)10^3}{10^{-3}} = 9677.4$$

$$Pr = \frac{C_p\mu}{K} = 4.2 \; KJ/KgK$$

Source data: Recently, Z hukauskas has given c,m ,ξ,n values.

For Re = 967704

C = 0.26 & m = 0.6

Nu$_D$ = hD/K = 0.193 (9677.4)*(6.774X10^{-3}) = 130

.h = 25380

5.3 Given a system with the transfer function Y(s)/X(s) = (T$_1$s+1)/(T$_2$s+1). Find Y(t) if X(t) is a unit step function. If T$_1$/T$_2$ = s. Sktech Y(t) Versus t/T$_2$. Show the numerical values of minimum, maximum and ultimate values that may occur during the transient. Check these using the initial value and final value theorems of chapter 4.

$$Y(s) = \frac{T_1 s + 1}{T_2 s + 1}$$

X(s) = unit step function = 1 X(s) = 1/s

$$Y(s) = \frac{T_1 s + 1}{s(T_2 s + 1)} = \frac{A}{s} + \frac{B}{iT_2 s}$$

A = 1   B = T₁ - T₂

$$Y(s) = \frac{1}{s} + \frac{T_1 - T_2}{1 + T_2 s}$$

$$Y(t) = 1 + \frac{T_1 - T_2}{T_2} e^{-t/T_2}$$

If T₁/T₂ = s then

$$Y(t) = 1 + 4e^{-t/T_2}$$

Let t/T₂ = x then $Y(t) = 1 + 4e^{-x}$

Using the initial value theorem and final value theorem

$$\operatorname*{Lim}_{T \to 0} Y(T) = \operatorname*{Lim}_{S \to \infty} sY(s)$$

$$= \operatorname*{Lim}_{S \to \infty} \frac{T_1 s + 1}{T_2 s + 1} = \operatorname*{Lim}_{S \to \infty} \frac{T_1 + \dfrac{1}{s}}{T_2 + \dfrac{1}{s}} = \frac{T_1}{T_2} = 5$$

$$\operatorname*{Lim}_{T \to 0} Y(T) = \operatorname*{Lim}_{S \to \infty} sY(s) = \operatorname*{Lim}_{S \to 0} \frac{T_1 s + 1}{T_2 s + 1} = 1$$

Figure:

Problem 5.3
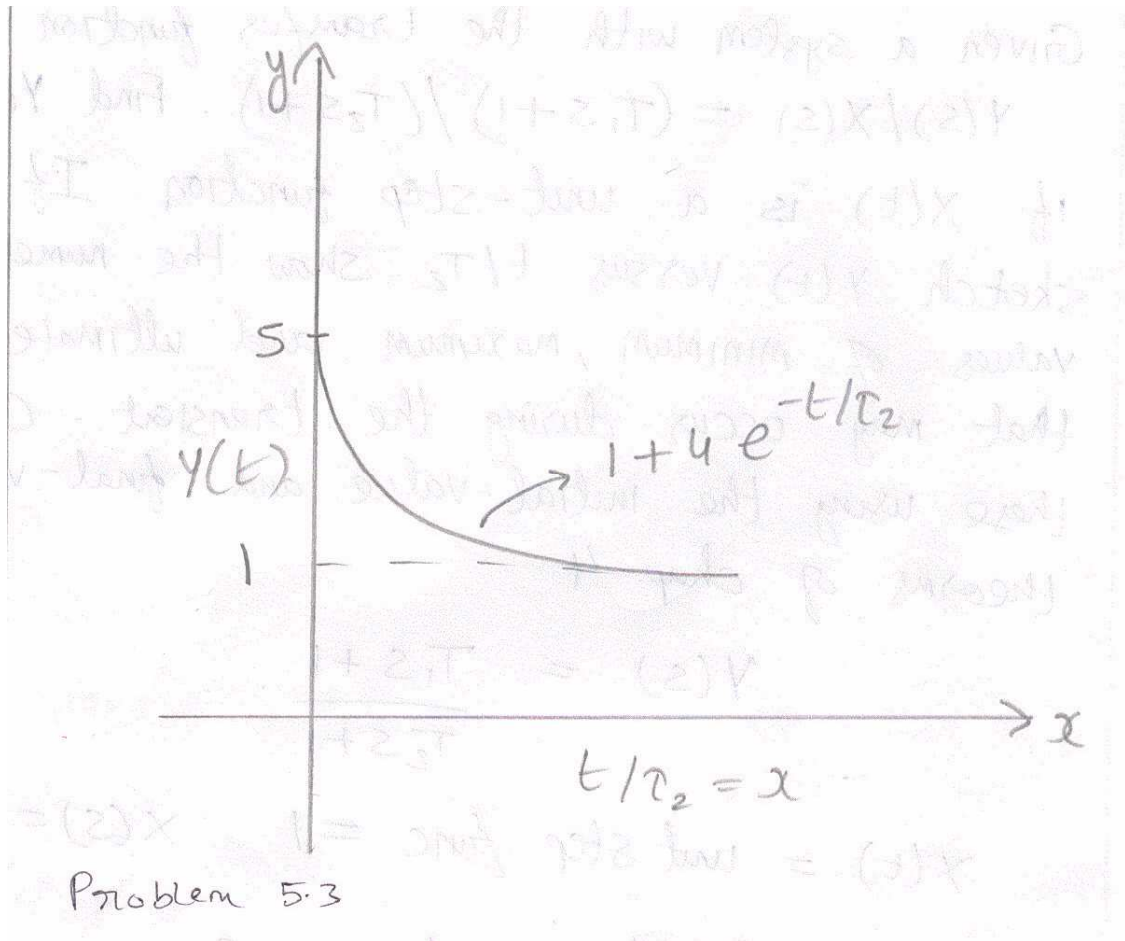
5.4 A thermometer having first order dynamics with a time constant of 1 min is placed in a temperature bath at 100 deg F. After the thermometer reaches steady state, it is suddenly placed in bath at 100 deg F at t = 0 and left there for 1 min after which it is immediately returned to the bath at 100 deg F.

(a) draw a sketch showing the variation of the thermometer reading with time.

(b) calculate the thermometer reading at t = 0.5 min and at t = 2.0 min

$$\frac{Y(s)}{X(s)} = \frac{1}{s+1} \ (\tau = 1 \text{ min})$$

$$(s) = 10\left[\frac{1}{s} - \frac{e^{-s}}{s}\right]$$

$$Y(s) = 10\left[\frac{1-e^{-s}}{s}\right]$$
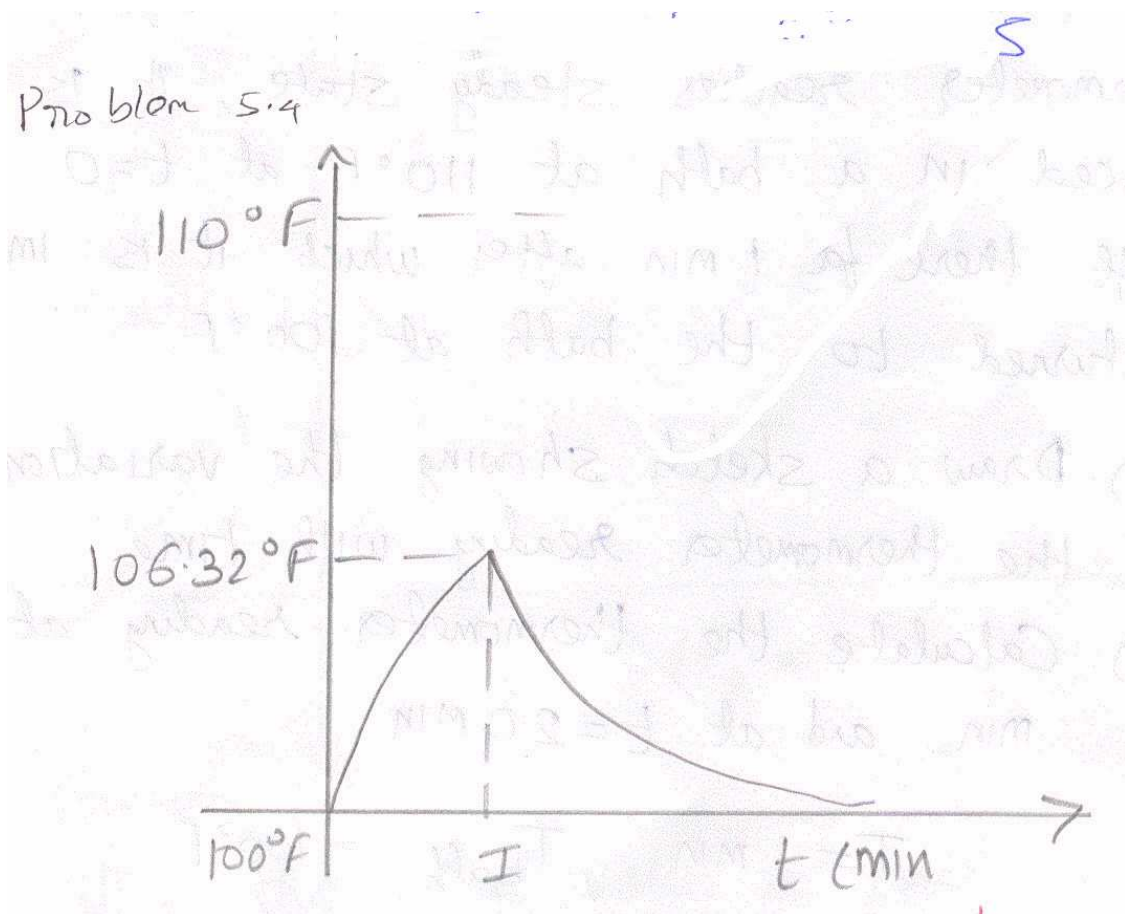
$$Y(s) = 10\left[\frac{1}{s(s+1)}\frac{-e^{-s}}{s(s+1)}\right]$$

$$Y(t) = 10(1-e^{-t})\ t < 1$$

$$Y(t) = 10\left(\ (1-e^{-t})-(1-e^{-(t-1)})\ \right)t \geq 1$$

At t = 0.5 T = 103.93

At = 2 T =102.325

Problem 5.4

5.5 Repeat problem 5.4 if the thermometer is in 110 deg F for only 10 sec.

If thermometer is in 110 deg F bath for only 10 sec

$$T = 110 - 10e^{-t/60}$$

$$0 < t < 10 \text{ sec } \& \ T = 60 \text{ sec}$$

$$T(t = 10 \sec) = 101.535$$

$$T = 100 + 1.535 e^{-(t-10)/60} \quad t > 10 \text{ sec}$$

T(t=30sec) = 101.099 deg F

T(t=120sec) = 100.245 deg F
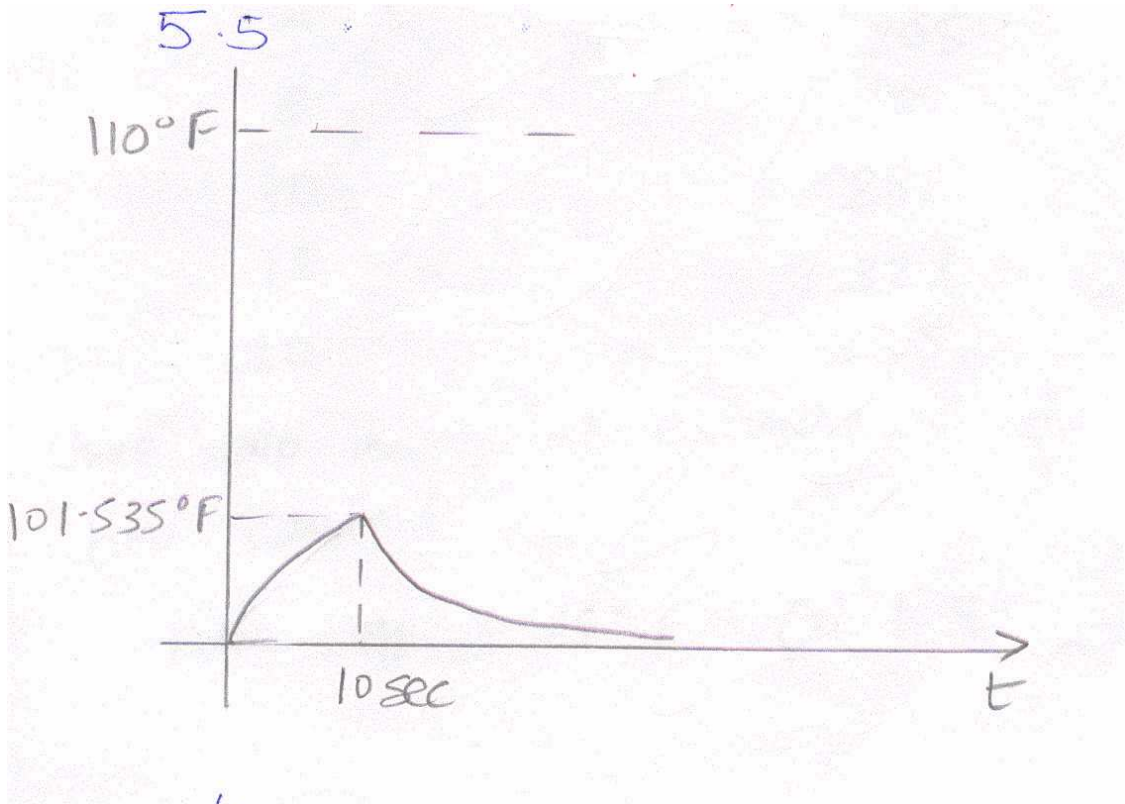
5.6 A mercury thermometer which has been on a table for some time,is registering the room temperature ,758 deg F. Suddenly, it is placed in a 400 deg F oil bath. The following data are obtained for response of the thermometer

Time (sec)	Temperature, Deg F
0	75
1	107
2.5	140
5	205
8	244
10	282
15	328
30	385

Give two independent estimates of the thermometer time constant.

$$\tau = \frac{t}{\ln\left(\dfrac{325}{400 - T}\right)}$$

From the data , average of 9.647,11.2,9.788,10.9,9.87,9.95, and 9.75 is 10.16 sec.

5.7 Rewrite the sinusoidal response of first order system (eq 5.24) in terms of a cosine wave. Re express the forcing function equation (eq 5.19) as a cosine wave and compute the phase difference between input and output cosine waves.

$$Y(s) = \frac{1}{\tau s + 1}(s) = \frac{A\omega}{s^2 + \omega^2} \frac{\left(\dfrac{1}{\tau}\right)}{s + \dfrac{1}{\tau}}$$

splitting into partial fractions then converting to laplace transforms

$$Y(t) = \frac{A\alpha\omega\tau}{\tau^2\omega^2 + 1}e^{-t/\tau} + \frac{A}{\sqrt{\tau^2\omega^2 + 1}}\sin(\omega t + \phi)$$

where $\phi = \tan^{-1}(\omega\tau)$

As t $\rightarrow\infty$

$$Y(t)\big|s = \frac{A}{\sqrt{\tau^2\omega^2 + 1}}\sin(\omega t + \phi) = \frac{A}{\sqrt{\tau^2\omega^2 + 1}}\cos\left(\omega t - \left(\frac{\pi}{2} - \phi\right)\right)$$

$$Y(t) = A\sin(\omega t + \phi) = A\cos\left(\frac{\pi}{2} - \omega t\right)$$

$$Y(t) = A\cos\left(\omega t - \frac{\pi}{2}\right)$$

The phase difference $= \phi - \dfrac{\pi}{2} - \left(-\dfrac{\pi}{2}\right) = |\phi|$

5.8 The mercury thermometer of problem 5.6 is allowed to come to equilibrium in the room temp at 75 deg F. Then it is immersed in a oil bath for a length of time less than 1 sec and quickly removed from the bath and re exposed to 75 deg F ambient condition. It may be estimated that the heat transfer coefficient to the thermometer in air is 1/5th that in oil bath. If 10 sec after the thermometer is removed from the bath it reads 98 Deg F. Estimate the length of time that the thermometer was in the bath.

t < 1 sec $\quad T_1 = 400 - 325e^{-t_1/\tau}$

Next it is removed and kept in 75 Deg F atmosphere

Heat transfer co-efficient in air = 1/5 heat transfer co-efficient in oil

$h_{air} = 1/5 \ h_{oil}$

$\tau = \dfrac{mC}{hA} \quad \tau_{oil} = 10\,\text{sec}$

$\qquad\qquad \tau_{air} = 50\,\text{sec}$

$T_F = 75 + (T_1 - 75)e^{-t_2/50}$

$T_F = Final\ Temp = 98\deg C$

$98 = 75 + (325 \ -325e^{-t_1/10})e^{-10/50}$

$e^{-t/10} = 0.91356$

t$_1$ = 0.904 sec.

5.9 A thermometer having a time constant of 1 min is initially at 50 deg C. it is immersed in a bath maintained at 100 deg C at t = 0. Determine the temperature reading at 1.2 min.

τ = 1 min for a thermometer initially at 50 deg C.

Next it is immersed in bath maintained at 100 deg C at t = 0

At t = 1.2

$$Y(t) = A(1 - e^{-t/\tau})$$

$$Y(1.2) = 50(1 - e^{-1.2/1}) + 50$$

$$Y(1.2) = 84.9 \text{ deg } C$$

5.10 In Problem No 5.9 if at, t = 1.5 min thermometer having a time constant of 1 minute is initially at 50 deg C.It is immersed in a bath maintained at 100 deg C at t = 0.Determine the temperature reading at t = 1.2 min.

At t = 1.5

$$Y(1.5) = 88.843°C$$

Max temperature indicated = 88.843 deg C

AT t = 20 min

$$T = 88.843 - 13.843(1 - e^{-18.8/1})$$

T = 75 Deg C.

5.11 A process of unknown transfer function is subjected to a unit impulse input. The output of the process is measured accurately and is found to be represented by the function Y(t) = t e^{-t}. Determine the unit step response in this process.

X(s) = 1 Y(t) = te^{-t}

$$Y(s) = \frac{1}{(s+1)^2}$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)^2}$$

For determining unit step response

$$Y(s) = \frac{1}{(s+1)^2}$$

$$Y(s) = \frac{1}{(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

A = 1   B = -1   C = -1

$$Y(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$Y(t) = 1 - e^{-t} - te^{-t}$$

Response of first order system in series

7.1 Determine the transfer function H(s)/Q(s) for the liquid   level shown in figure P7-7. Resistance R_1 and R_2 are linear. The flow rate   from   tank 3 is maintained constant at b by means of a pump ; the flow rate   from tank 3 is independent of head h. The tanks are non interacting.

Solution :

FIGURE P7-1
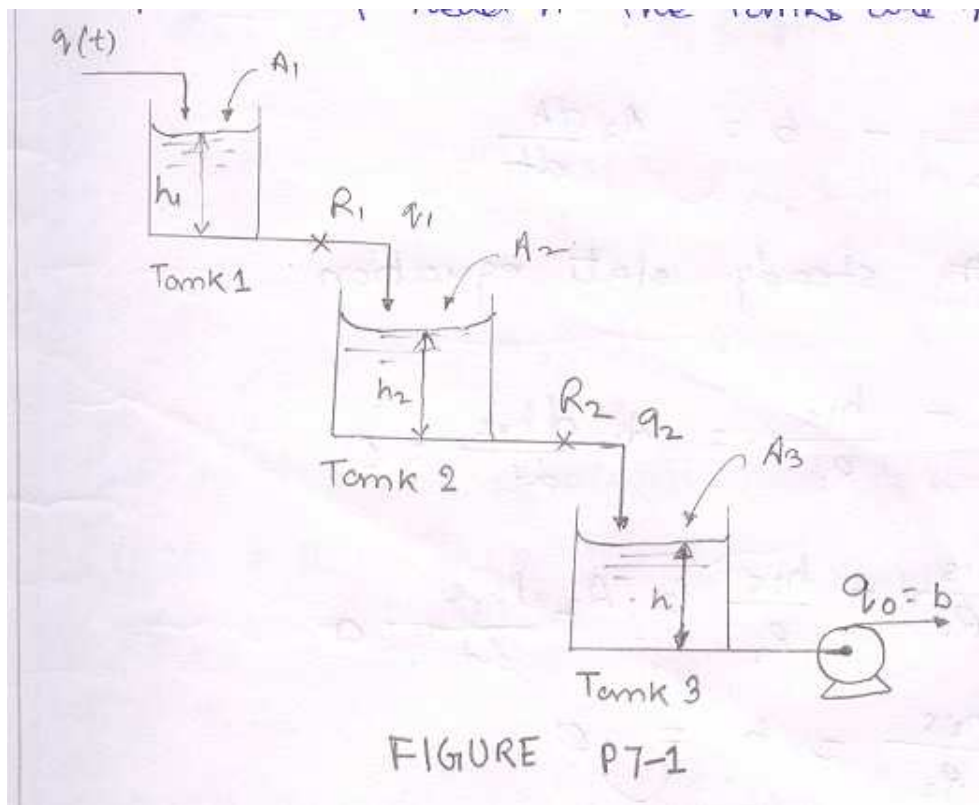
A balance on tank 1 gives

$$q - q_1 = A_1 \frac{dh_1}{dt}$$

where h1 = height of the liquid level in tank 1

similarly balance on the tank 2 gives

$$q_1 - q_2 = A_2 \frac{dh_2}{dt}$$

and balance   on tank 3 gives

$$q_2 - q_0 = A_3 \frac{dh}{dt}$$

here $q_1 = \dfrac{h_1}{R_1}$ $q_2 = \dfrac{h_2}{R_2}$ $q_0 = b$

So we get

$$q - \frac{h_1}{R_1} = A_1 \frac{dh_1}{dt}$$

$$\frac{h_1}{R_1} - \frac{h_2}{R_2} = A_2 \frac{dh_2}{dt}$$

$$\frac{h_2}{R_2} - b = A_3 \frac{dh}{dt}$$

writing the steady state equation

$$q_S - \frac{h_{1S}}{R_1} = A_1 \frac{dh_{1S}}{dt} = 0$$

$$\frac{h_{1S}}{R_1} - \frac{h_{2S}}{R_2} = A_2 \frac{dh_{2S}}{dt}$$

$$\frac{h_{2S}}{R_2} - b = 0$$

Subtracting and writing in terms of deviation

$$Q - \frac{H}{R_1} = A_1 \frac{dH_1}{dt}$$

$$\frac{H_1}{R_1} - \frac{H_2}{R_2} = A_1 \frac{dH_2}{dt}$$

$$\frac{H_2}{R_2} = A_3 \frac{dH}{dt}$$

where   Q = q −q$_S$

   H$_1$= h$_1$-h$_{1S}$

   H$_1$= h$_2$-h$_{2S}$

   H =   h - h$_S$

Taking Laplace transforms

$$Q(s) - \frac{H_1(s)}{R_1} = A_1 s \, H_1(s) \text{---------(1)}$$

$$\frac{H_1(s)}{R_1} - \frac{H_2(s)}{R_2} = A_2 s \, H_2(s) \text{ --------(2)}$$

$$\frac{H_2(s)}{R_2} = A_3 \, s H(s) \text{----------(3)}$$

We have three equations   and 4 unknowns(Q(s),H(s),H$_1$(s) and H$_2$(s). So   we can   express one in terms of other.

From (3)

$$H_2(s) = \frac{H_2(s)}{R_1 A_3 s} \text{-------------(4)}$$

$$H_2(s) = \frac{R_2 H_1(s)}{R_1(\tau_2 s + 1)} \quad \text{where} \quad \tau_2 = R_2 A_2 \text{------------(5)}$$

From (1)

$$H_1(s) = \frac{R_1 Q(s)}{(\tau_1 s + 1)}, \quad \tau_1 = R_1 A_1 \text{---------(6)}$$

Combining equation 4,5,6

$$H(s) = \frac{Q(s)}{(A_3 s)(\tau_1 s + 1)(\tau_2 s + 1)}$$

$$\frac{H(s)}{Q(s)} = \frac{1}{(A_3 s)(\tau_1 s + 1)(\tau_2 s + 1)}$$

Above equation can be written as

i.e, if non interacting first order system are there in series then there overall transfer function is equal to the product of the individual transfer function in series.

7.2 The mercury thermometer in chapter 5 was considered to have all its resistance in the convective film surrounding the bulb and all its capacitance in the mercury. A more detailed analysis would consider both the convective resistance surrounding the bulb and that between the bulb and mercury. In addition , the capacitance of the glass bulb would be included.

Let
A_i = inside area of bulb for heat transfer to mercury.
A_o = outside area of bulb, for heat transfer from surrounding fluid.
.m = mass of the mercury in bulb.
m_b = mass of glass bulb.
C = heat capacitance of mercury.

Cb = heat capacity of glass bulb.
.hi = convective co-efficient between the bulb and the surrounding fluid.
.ho = convective co-efficient between bulb and surrounding fluid.
T = temperature of mercury.
Tb = temperature of glass bulb.
Tf = temperature of surrounding fluid.

Determine the transfer function between Tf and T. what is the effect of bulb resistance and capacitance on the thermometer response? Note that the inclusion of the bulb results in a pair of interacting systems, which give an overall transfer function different from that of Eq (7.24)

Writing the energy balance for change in term of a bulb and mercury respectively
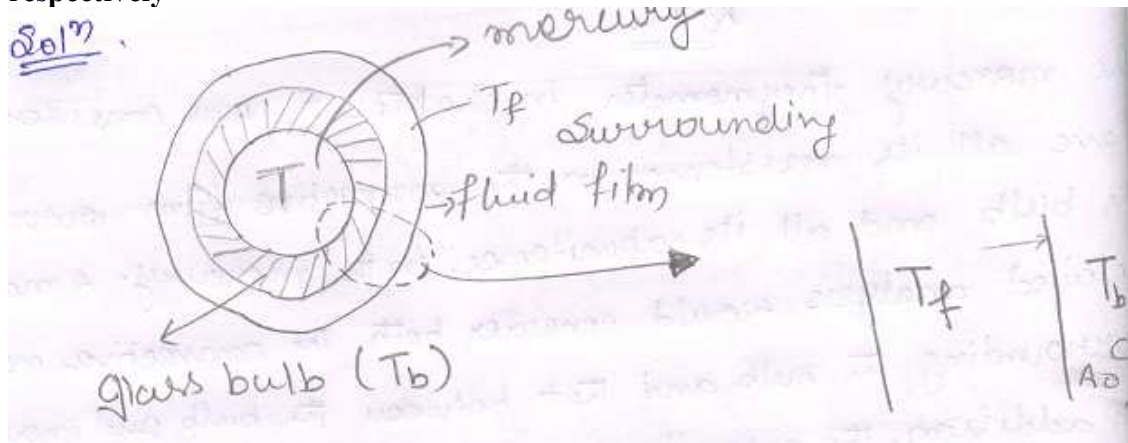
Input - output = accumulation

$$h_0 A_0 (T_f - T_b) - h_i A_i (T_b - T) = m_b C_b \frac{dT_b}{dt}$$

$$h_i A_i (T_b - T) - 0 = m\, C\, \frac{dT}{dt}$$

Writing the steady state equation

$$h_0 A_0 (T_{fs} - T_{bs}) - h_i A_i (T_{bs} - T_s) = m_b C_b \frac{dT_{bs}}{dt} = 0$$

$$h_i A_i (T_{bs} - T_s) = 0$$

Where subscript s denoted values at steady subtracting and writing these equations in terms of deviation variables.

$$h_0 A_0 (T_f - T_b) - h_i A_i (T_b - T_m) = m_b C_b \frac{dT_b}{dt}$$

$$h_i A_i (T_b - T_m) - 0 = m \, C \, \frac{dT_m}{dt}$$

Here $T_F = T_f - T_{fS}$
$T_B = T_b - T_{bS}$
$Tm = T - T_S$

Taking laplace transforms

$$h_0 A_0 (T_F(s) - T_B(s)) - h_i A_i (T_B - T_m) = m_b C_b T_B(s) \text{----(1)}$$

And $h_i A_i (T_B(s) - T_m(s)) = mC \, s T_B(s) \text{------(2)}$

$$= \, h_0 A_0 (T_F(s) - T_B(s)) - mCST_m(s) = m_b C_b s T_B(s)$$

From (2) we get

$$T_B(s) = T_m(s) \left[\frac{mC}{h_i A_i} s + 1 \right] = T_m(s) \, (\tau_i s + 1)$$

Where $\tau_i = \dfrac{mC}{h_i A_i}$

Putting it into (1)

$$T_F(s) - \, T_m(s) \left[(\tau_i s + 1))(\tau_0 s + 1) + \frac{mC}{h_0 A_0} s \right] = 0$$

$$= T_F(s) = T_m(s)\left[(\tau_i s + 1))(\tau_0 s + 1) + \frac{mC}{h_0 A_0} s\right]$$

$$= \frac{T_m(s)}{T_F(s)} = \frac{1}{\tau_i \tau_0 s^2 + (\tau_i + \tau_0 + \frac{mC}{h_0 A_0})s + 1}$$

$$= \frac{T_m(s)}{T_F(s)} = \frac{1}{\tau_i \tau_0 s^2 + (\tau_i + \tau_0 + \frac{mC}{h_0 A_0})s + 1}$$

Or we can write

$$\frac{T(s)}{T_f(s)} = \frac{1}{\tau_i \tau_0 s^2 + (\tau_i + \tau_0 + \frac{mC}{h_0 A_0})s + 1}$$

$$\tau_i = \frac{mC}{h_i A_i} \quad \text{and} \quad \tau_0 = \frac{m_b C_b}{h_0 A_0}$$

We see that a loading term mC/h_oA_o is appearing in the transfer function. The bulb resistance and capacitance is appear in τ_0 and it increases the delay i.e Transfer lag and response is slow down.

7.3 There are N storage tank of volume V Arranged so that when water is fed into the first tank into the second tank and so on. Each tank initially contains component A at some concentration Co and is equipped with a perfect stirrer. A time zero, a stream of zero concentration is fed into the first tank at volumetric rate q. Find the resulting concentration in each tank as a function of time.

Solution:

Soln
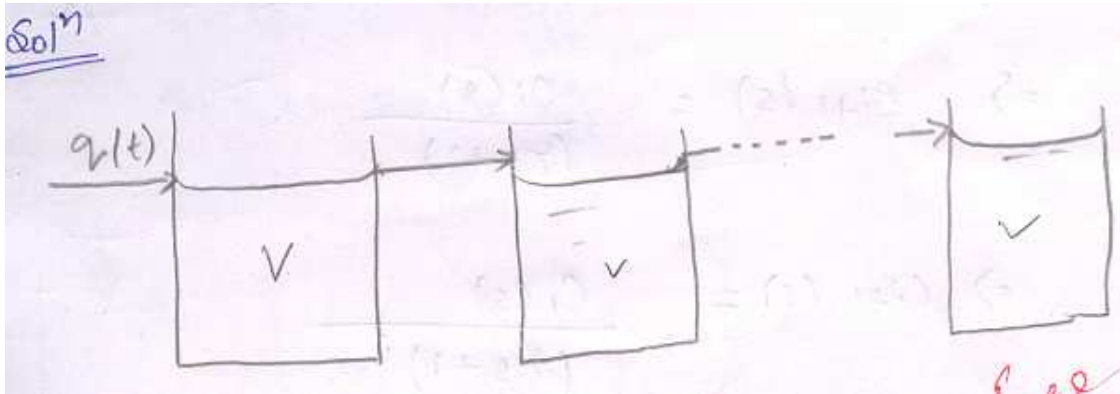
. i^{th} tank balance

$$qC_{i-1} - qC_i = V\frac{dC_i}{dt}$$

$$qC_{(i-1)s} - qC_{is} = 0$$

$$C_{(i-1)} - C_i = \frac{V}{q}\frac{dC_i}{dt}$$

$$\left(\tau = \frac{V}{q}\right)$$

Taking lapalce transformation

$$C_{(i-1)}(s) - C_i(s) = \tau s Ci(s)$$

$$C_{(i-1)}(s) = (1 + \tau s)Ci(s)$$

$$\frac{C_i(s)}{C_{i-1}(s)} = \frac{1}{1 + \tau s}$$

Similarly

$$\frac{C_i(s)}{Co(s)} = \frac{C_1(s)}{C_0(s)} \times \frac{C_2(s)}{C_1(s)} \times - - - - - - - - \frac{C_{i-1}(s)}{C_{i-2}(s)} \times \frac{C_i(s)}{C_i(s)} = \frac{1}{(1 + \tau s)i}$$

Or

$$\frac{C_N(s)}{Co(s)} = \frac{1}{(1+\tau s)^N}$$

$$C_N(s) = \frac{-C_0}{s(1+\tau s)^N}$$

$$C_N(s) = -C_0 \left[\frac{1}{s} - \frac{\tau}{(1+\tau s)^N} - \frac{\tau}{(1+\tau s)^{N-1}} ------- \frac{\tau}{1+\tau s} \right]$$

$$C_N(t) = -C_0 \left[1 - \frac{e^{-\frac{t}{\tau}}}{\tau^{N-1}} \cdot \frac{t^{N-1}}{(N-1)!} - \frac{e^{-\frac{t}{\tau}}}{\tau^{N-2}} \cdot \frac{t^{N-2}}{(N-2)!} ----- e^{-\frac{t}{\tau}} \right]$$
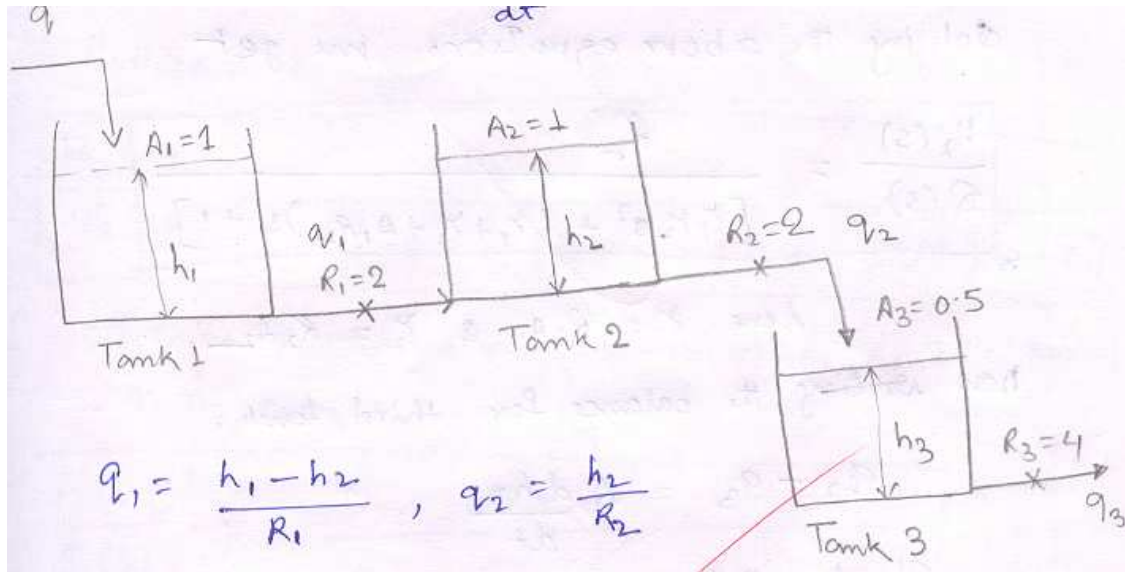
$$C_N(t) = -C_0 \left[1 - e^{-\frac{t}{\tau}} \left(-\frac{\left(\frac{t}{\tau}\right)^{N-1}}{(N-1)!} + \cdot \frac{\left(\frac{t}{\tau}\right)^{N-2}}{(N-2)!} ----- +1 \right) \right]$$

7.4 (a) Find the transfer functions H_2/Q and H_3/Q for the three tank system shown in Fig P7-4 where H_1, H_3 and Q are deviation variables. Tank 1 and Tank 2 are interacting.

7.4(b) For a unit step change in q (i.e $Q = 1/s$); determine $H_3(0)$, $H_3(\infty)$ and sketch $H_3(t)$ vs t.

Solution :

   Writing heat balance equation for tank 1 and tank 2

$$q - q_1 = A_1 \frac{dh_1}{dt}$$

$$q_1 - q_2 = A_2 \frac{dh_2}{dt}$$

$$q_1 = \frac{h_1 - h_2}{R_1} \quad q_2 = \frac{h_2}{R_2}$$

Writing the steady state equation

$$q_s - q_{1s} = 0$$

$$q_{1s} - q_{2s} = 0$$

Writing the equations in terms of deviation variables

$$Q - Q_1 = A_1 \frac{dH_1}{dt}$$

$$Q_1 - Q_2 = A_2 \frac{dH_2}{dt}$$

$$Q_1 = \frac{H_1 - H_2}{R_1} \quad Q_2 = \frac{H_2}{R_2}$$

Taking laplace transforms

$$Q(s) - Q_1(s) = A_1 s H_1(s)$$

$$Q_1(s) - Q_2(s) = A_1 s H_2(s)$$

$$R_1 Q_1(s) = H_1(s) - H_2(s)$$

$$R_2 Q_2(s) = H_2(s)$$

Solving the above equations we get

$$\frac{H_2(s)}{Q(s)} = \frac{R_2}{\left[\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2 + A_1 R_2) s + 1 \right]}$$

Here $\tau_1 = R_1 A_1$

$$\tau_2 = R_2 A_2$$

Now writing the balance   for third tank

$$q_2 - q_3 = A_3 \frac{dh_3}{dt}$$

Steady state equation

$$q_{2S} - q_{3S} = 0$$

$$q_3 = \frac{h_3}{R_3}$$

$$Q_2 - \frac{H_3}{R_3} = A_3 \frac{dh_3}{dt}$$

Taking laplace transforms

$$Q_2(s) - \frac{H_3(s)}{R_3} = A_3 s H(s)$$

$$Q_2(s) = \frac{H_3(s)}{R_3}(\tau_3 s + 1) \quad \text{where } \tau_3 = R_3 A_3$$

From equation 1,2,3,4 and 5 we got

$$\frac{Q_s(s)}{Q(s)} = \frac{1}{\left[\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2 + A_1 R_2)s + 1\right]}$$

Putting it in equation 6

$$\frac{H_3(s)}{Q(s)} = \frac{R_3}{\left[\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2 + A_1 R_2)s + 1\right](\tau_3 s + 1)}$$

Putting the numerical values of R₁,R₂ and R₃ and A₁,A₂,A₃

$$\frac{H_3(s)}{Q(s)} = \frac{4}{\left[4s^2 + 6s + 1\right](2s + 1)}$$

$$\frac{H_2(s)}{Q(s)} = \frac{2}{\left[4s^2 + 6s + 1\right]}$$

Solution (b)

$$Q(s) = \frac{1}{s}$$

$$H_3(s) = \frac{1}{s} \frac{4}{\left[4s^2 + 6s + 1\right](2s+1)}$$

From initial value theorem

$$H_3(0) = \lim_{S \to \infty} sH_3(s)$$

$$= \lim_{S \to \infty} \frac{4}{(2s+1)(4s^2 + 6s + 1)}$$

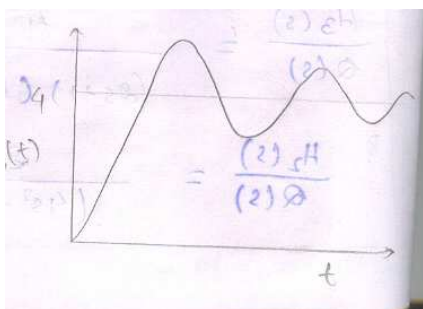$$= \lim_{S \to \infty} \frac{4}{(2s+1)s^3\left(4 + \dfrac{6}{s} + \dfrac{1}{s^2}\right)}$$

$$H_3(0) = 0$$

From final value theorem

$$H_3(\infty) = \lim_{S \to 0} sH_3(s)$$

$$= \lim_{S \to 0} \frac{4}{(2s+1)(4s^2 + 6s + 1)}$$
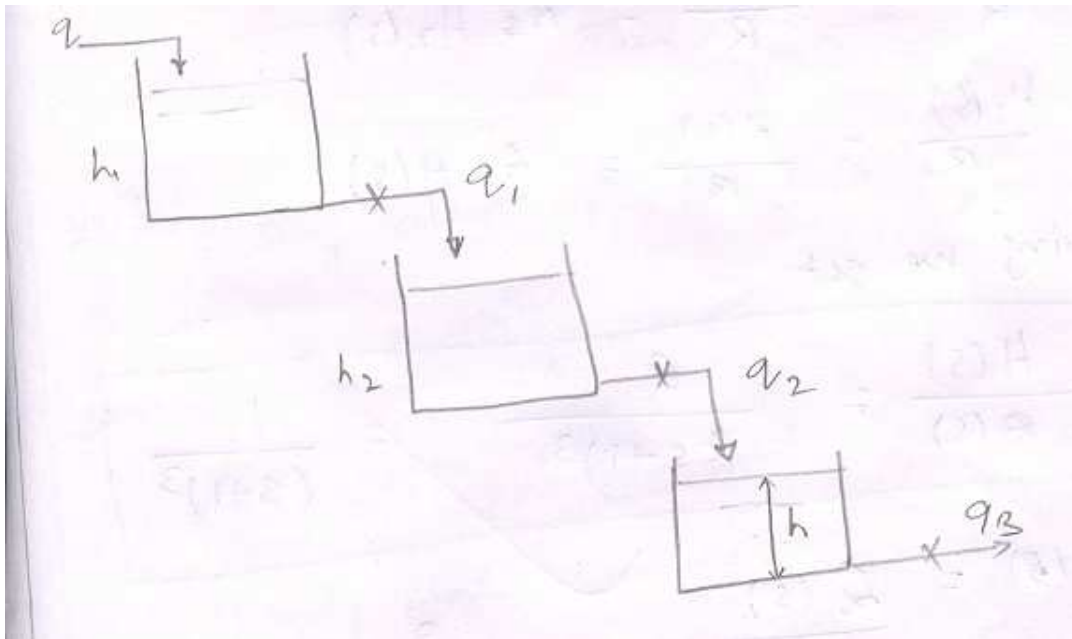
$$H_3(\infty) = 4$$

7.5 Three identical tanks are operated in series in a non-interacting fashion as shown in fig P7.5 . For each tank R=1, τ = 1. If the deviation in flow rate to the first tank in an impulse function of magnitude 2, determine

(a) an expression for H(s) where H is the deviation in level in the third tank.

  (b) sketch the response H(t)

  (c) obtain an expression for H(t)

solution :

   writing energy balance equation for all tanks

$$q - q_1 = A \frac{dh_1}{dt}$$

$$q_1 - q_2 = A \frac{dh_2}{dt}$$

$$q_2 - q_3 = A\, \frac{dh}{dt}$$

$$q_1 = \frac{h_1}{R} \quad q_2 = \frac{h_2}{R} \quad q_3 = \frac{h}{R}$$

So we get

$$q_S - q_{1S} = 0$$

$$q_{1S} - q_{2S} = 0$$

$$q_{2S} - q_{3S} = 0$$

writing in terms of deviation variables and taking laplace transforms

$$Q(s) - \frac{H_1(s)}{R} = A_S\, H_1(s)$$

$$\frac{Q_1(s)}{R} - \frac{H_2(s)}{R} = A_S\, H_2(s)$$

$$\frac{H_2(s)}{R} - \frac{H(s)}{R} = A_S\, H(s)$$

solving we get

$$\frac{H(s)}{Q(s)} = \frac{R}{(\tau s + 1)^3} = \frac{1}{(s+1)^3}$$

$$H(s) = \frac{Q(s)}{(\tau s + 1)^3} = \frac{2}{(s+1)^3}$$
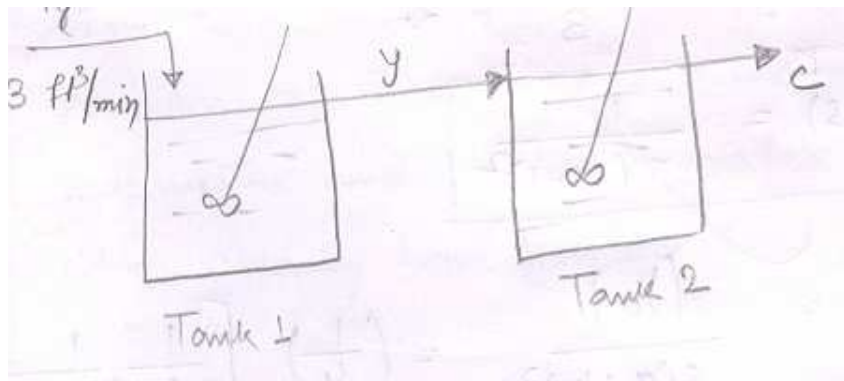
$$H(t) = L^{-1}\{H(s)\} = 2\frac{t^2}{2}e^{-t}$$

$$H(t) = t^2 e^{-t}$$

$$\frac{dH(t)}{dt} = 2te^{-t} - te^{-t} = 0$$

$$= 2t = t^2$$

at t = 2 max will occur.

7.6 In the two- tank mixing process shown in fig P7.6 , x varies from 0 lb salt/ft$_3$ to 1 lb salt/ft^3 according to step function. At what time does the salt concentration in tank 2 reach 0.6 lb/ ft^3 ? The hold up volume of each tank is 6 ft^3.

Solution

Writing heat balance equation for tank 1 and tank 2

$$q_x - q_y = V\frac{dy}{dt}$$

$$q_y - q_c = V \frac{dl}{dt}$$

steady state equation

$$q_{xs} - q_{ys} = 0$$

$$q_{ys} - q_{cs} = 0$$

writing in terms of deviation variables and taking laplace transforms

$$X(s) - Y(s) = \frac{V}{q} s Y(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{\left(\dfrac{V}{q} s + 1\right)} = \frac{1}{\tau s + 1} \; ; \tau = \frac{V}{q}$$

$$C(s) = \frac{Y(s)}{(\tau s + 1)} = \frac{X(s)}{(\tau s + 1)^2}$$

$$\frac{C(s)}{X(s)} = \frac{1}{(\tau s + 1)^2}$$

$$X(s) = \frac{1}{s}$$

$$\tau = \frac{V}{q} = \frac{6}{3} = 2$$

$$C(s) = \frac{X(s)}{s(2s + 1)^2}$$

$$C(s) = \frac{(1/4)}{s(s + \frac{1}{2})^2}$$

$$C(s) = \left(\frac{1}{4}\right) \left\{ \frac{1}{s} \frac{1}{(s + \frac{1}{2})^2} - \right\}$$

$$C(s) = \frac{1}{s} - \frac{\left(\frac{1}{2}\right)}{\left[1 + \frac{1}{2}\right]^2} - \frac{1}{\left(s + \frac{1}{2}\right)}$$

$$C(t) = 1 - \frac{1}{2} t e^{-\frac{t}{2}} - e^{-\frac{t}{2}}$$

$$C(t) = 0.61 \ lb \ salt \ / \ ft^3$$

t = 4.04 min

7.7 Starting from first principles, derive the transfer functions $H_1(s)/Q(s)$ and $H_2(s)/Q(s)$ for the liquid level system shown in figure P7.7. The resistance are linear and $R_1 = R_2 = 1$. Note that two streams are flowing from tank 1, one of which flows into tank 2. You are expected to give numerical values of the parameters and in the transfer functions and to show clearly how you derived the transfer functions.
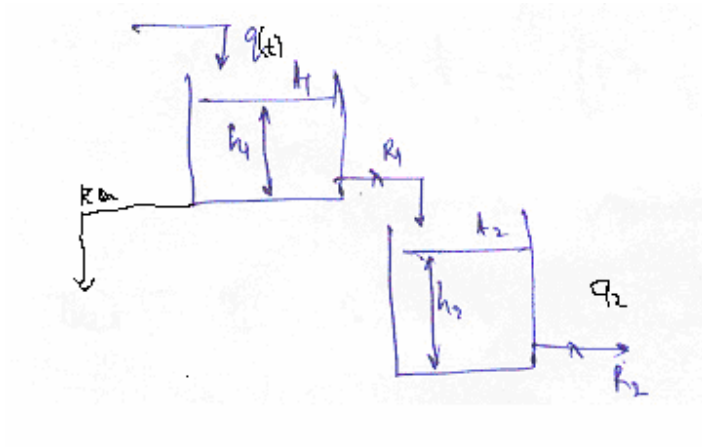
Writing heat balance equation for tank 1

$$q - q_a - q_1 = A_1 \frac{dh_1}{dt}$$

$$q_1 = \frac{h_1}{R_1} \quad q_a = \frac{h_1}{R_a}$$

$$= q - \frac{h_1}{R_a} - \frac{h_1}{R_1} = A_1 \frac{dh_1}{dt}$$

writing the balance equation for tank 2

$$q_1 - q_2 = A_2 \frac{dh_2}{dt}$$

$$\frac{h_1}{R_1} - \frac{h_2}{R_2} = A_2 \frac{dh_2}{dt}$$

writing steady state equations

$$q_s - \frac{h_s}{R_a} - \frac{h_1 s}{R_1} = 0$$

$$\frac{h_1s}{R_1} - \frac{h_2s}{R_2} = 0$$

writing the equation in terms of deviation variables

$$Q - H_1\left(\frac{1}{R_a} + \frac{1}{R_1}\right) = A_1 \frac{dH_1}{dt}$$

$$\frac{H_1}{R_1} - \frac{H_2}{R_2} = A_2 \frac{dH_2}{dt}$$

taking laplace transforms

$$Q(s) - H_1(s)\left(\frac{R_1 + R_2}{R_1 R_a}\right) = A_{1S} \, H_1 s \text{-----------(1)}$$

and $\quad \dfrac{H_1(s)}{R_1} - \dfrac{H_2(s)}{R_2} = A_2 s \, H_2(s) \text{-----------(2)}$

from (1) we get

$$\frac{H_1(s)}{Q(s)} = \frac{1}{\left[A_1 s + \dfrac{R_1 + R_a}{R_1 R_a}\right]}$$

$$\frac{H_1(s)}{Q(s)} = \frac{\left[\dfrac{R_1 R_a}{R_1 + R_a}\right]}{\left[\dfrac{R_1 R_a A_1}{R_1 + R_a} s + 1\right]}$$

$$\frac{H_1(s)}{Q(s)} = \frac{\left[\dfrac{R_1 R_a}{R_1 + R_a}\right]}{[\tau_1 s + 1]} \quad ; \quad \tau_1 = \frac{R_1 R_a A_1}{R_1 + R_a}$$

and from (2) we get

$$\frac{H_1(s)}{Q(s)} = \frac{\left[\dfrac{R_2 R_a}{R_1 + R_a}\right]\left(\dfrac{R_2}{R_1}\right)}{[\tau_1 s + 1](\tau_2 s + 1)} \quad \tau_2 = R_2 A_2$$

putting the numerical values of parameters

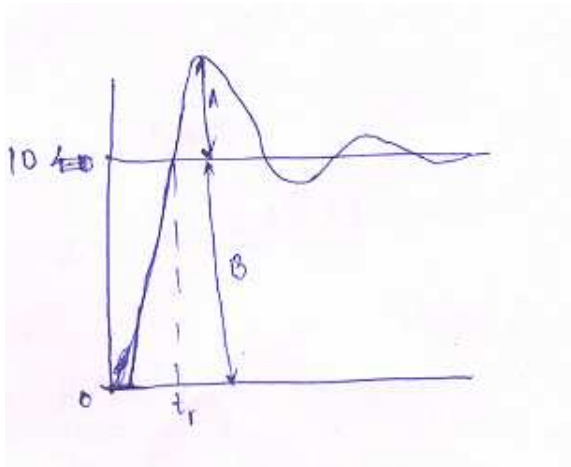$$\frac{H_1(s)}{Q(s)} = \frac{\left(\dfrac{2}{3}\right)}{\left(\dfrac{4}{3} s + 1\right)}$$

$$\frac{H_2(s)}{Q(s)} = \frac{\left(\dfrac{2}{3}\right)}{\left(\dfrac{4}{3} s + 1\right)(s + 1)}$$

8.1 A step change of magnitude 4 is introduced into a system having the transfer

$$\frac{Y(s)}{X(s)} = \frac{10}{s^2 + 1.6s + 4}$$

Determine   **(a) % overshoot**
                    (b)Rise time
                    (c)Max value of Y(t)
                    (d)Ultimate value of Y(t)
                    (e) Period of Oscillation.

Given $X(s) = \dfrac{4}{s}$ $Y(s) = \dfrac{40}{s(s^2 + 1.6s + 4)}$

The transfer function is $\dfrac{Y(s)}{X(s)} = \dfrac{10 \times 0.25}{0.2(s^2) + (\dfrac{1.6}{4})s + 1)} = \dfrac{2.5}{0.25s^2 + 0.4s + 1)}$

$\tau^2 = 0.25$; $\tau = 0.5$

and $2\xi\tau = 0.4$ $\xi = \dfrac{0.4}{2(0.5)} = 0.4 \ (<1 = system\ is\ underdamped)$

we find ultimate value of $Y(t)$

$\underset{t\to\infty}{Lt}\ Y(t) = \underset{S\to 0}{Lt}\ sY(s) = \underset{S\to 0}{Lt}\ \dfrac{40\,s}{s(s^2 + 1.6 + 4)} = \dfrac{40}{4} = 10$

thus $B = 10$

now, from laplace transform tables

$Y(t) = 10\left[1 - \dfrac{1}{\sqrt{1-\xi^2}}\,e^{-\frac{\xi t}{\tau}}\sin(\alpha + \phi)\right]$

where $\alpha = \dfrac{\sqrt{1-\xi^2}}{\tau}$, $\phi = \tan^{-}\left\|\dfrac{\sqrt{1-\xi^2}}{\xi}\right\|$

(a) Over shoot $= \dfrac{A}{B} = \exp\left(\dfrac{-\pi\xi}{\sqrt{1-\xi^2}}\right) = \exp\left[\dfrac{-\pi \times 0.4}{\sqrt{0.84}}\right] = 0.254$

thus % overshoot $= 25.4$

c)thus, max value of $Y(t) = A+B = B(0.254)+B$
$$= 2.54+10 = 12.54$$

e) Period of oscillation $= \dfrac{2\pi\tau}{\sqrt{1-\xi^2}} = 3.427$

b) For rise time, we need to solve

$$10\left[1 - \dfrac{1}{\sqrt{1-\xi^2}}\, e^{-\frac{\xi t}{\tau}}\sin(\alpha t + \phi)\right] = 10 \ \ for\, t = t_r$$

$= e^{-\frac{\xi\tau_r}{\tau}}\sin(\alpha t_r + \phi) = 0$

$= e^{-\frac{0.4\tau_r}{0.5}}\sin(1.833 t_r + 1.1589) = 0$

solving we get $t_r = 1.082$

thus

SOLUTION: % Overshoot = 25.4
 Rise time = 1.0842
 **Max Y(t) = 12.54**
 U(t) Y(t) = 10
 Period of oscillation = 3.427

Comment : we see that the Oscillation period is small and the decay ratio also small = system is efficiently under damped.

8.2 The tank system operates at steady state. At t = 0, 10 ft³ of water is added to tank 1. Determine the maximum deviation in level in both tanks from the ultimate steady state values, and the time at which each maximum occurs.

$A_1 = A_2 = 10 \text{ ft}^3$
$R_1 = 0.1\text{ft/cfm} \quad R_2 = 0.35\text{ft/cfm}.$
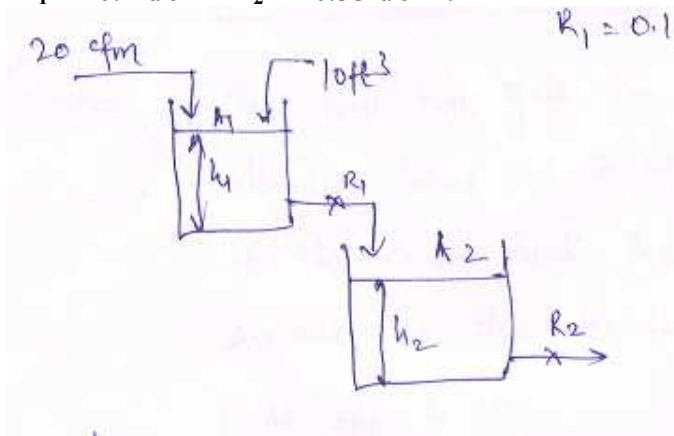
As the tanks are non interacting the transfer functions are

$$\frac{H(s)}{Q(s)} = \frac{K_1}{\tau_1 s + 1} = \frac{0.1}{(s+1)}$$

$$\frac{H_2(s)}{Q(s)} = \frac{R_2}{(\tau_1 s + 1)(\tau_2 s + 1)} = \frac{0.35}{(s+1)(3.5s+1)}$$

Now, an impulse of $\partial(t) = 10 \, ft^3 \, is \ provided$

$$Q(s) = 10 = H_1(s) = \frac{1}{s+1} = e^{-t}$$

and $H_2(s) = \dfrac{3.5}{(s+1)(3.5s+1)} = \dfrac{3.5}{3.5s^2 + 4.5s + 1}$

Now $\tau^2 = 3.5 = \tau = 1.871$

$$2\xi\tau = 4.5 = \xi = \frac{4.5}{2\tau} = 1.202$$

thus, this is an ovedamped system

Using fig8.5, for $\xi = 1.2$, we see that maximum is attained at

$\dfrac{t}{\tau} = 0.95, t = 1.776\,\text{min}$

And the maximum value is around $\tau_2 = 0.325\ Y_2(t) = 0.174$

$= H_2(t) = 0.174 \times 3.5 = 0.16\text{ft}$

thus max deviation is H_1 will be at $t = 0 = H_1 = 1$ ft

max deviation is H_2 will be at $t = 1.776$ min $= H_{2\text{Max}} = 0.61$ ft.

comment : the first tank gets the impulse and hence it max deviation turns out to be higher than the deviations for the second tank. The second tank exhibits an increase response ie the deviation increases, reaches the $H_{2\text{Max}}$ falls off to zero.

8.3 The tank liquid level shown operates at steady state when a step change is made in the flow to tank 1.the transaient response in critically damped, and it takes 1 min for level in second tank to reach 50 % of total change. If $A_1/A_2 = 2$,find R_1/R_2 . calculate τ for each tank. How long does it take for level in first tank to reach 90% of total change?
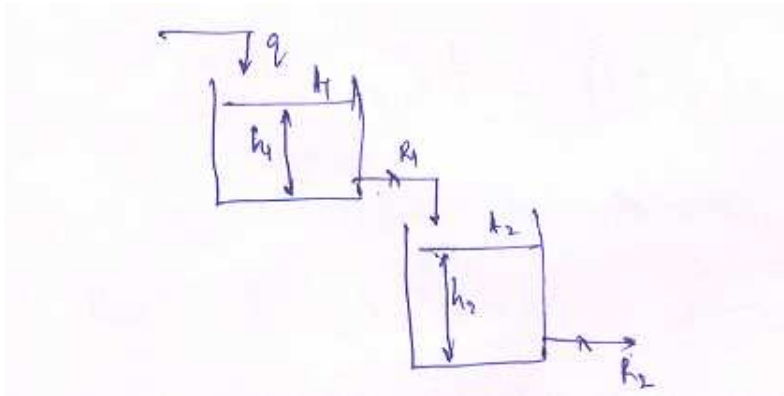
For the first tank, transfer function $\dfrac{H_1(s)}{Q(s)} = \dfrac{R_1}{\tau\ s + 1_1}$

For the second tank $\dfrac{H_2(s)}{Q(s)} = \dfrac{R}{(\tau_1 s + 1)(\tau_2 s + 1)}$

$$= \frac{H_2(s)}{Q(s)} = \frac{R_2}{\tau_1\tau_2 s^2 + (\tau_1 + \tau_2)s + 1}$$

$$Q(s) = \frac{1}{s} \; ; H_2(s) = \frac{1}{s} \frac{R_2}{\tau_1\tau_2 s^2 + (\tau_1 + \tau_2)s + 1}$$

$$\tau(parameter) = \sqrt{\tau_1\tau_2}$$

For $\tau(parameter) = \sqrt{\tau_1\tau_2}$

$$for\; \xi = 1, H_2(t) = R_2\left[1 - \left(1 + \frac{t}{\sqrt{\tau_1\tau_2}}\right)e^{-\frac{t}{\sqrt{\tau_1\tau_2}}}\right]$$

given, t $= 1$ for $\tau(parameter) = \sqrt{\tau_1\tau_2}$

$$H_2(t \to \infty) = R_2(1 - (0)) = R_2$$

$$= R_2\left(1 - \left(1 + \frac{1}{\sqrt{\tau_1\tau_2}}\right)e^{-\frac{1}{\sqrt{\tau_1\tau_2}}}\right) = \frac{R_2}{2} - I$$

also $2\xi\tau = \tau_1 + \tau_2$

$$\xi = 1 = \frac{\tau_1 + \tau_2}{2} = \tau = A_1R_1 = A_2R_2 = \frac{R_1}{R_2} = \frac{A_2}{A_1} = 0.5$$

from   I

$$1 - 0.5 = \left(1 + \frac{1}{\tau}\right)e^{-\frac{1}{\tau}}$$

$$8.3) H_1(s) = \frac{R_1}{s(\tau_1 s + 1)}; \ H_1(t) = R_1(1 - e^{-\frac{t}{\tau_1}})$$

$$0.94(t \to \infty) = R_1(1 - e^{-\frac{t}{\tau_1}})$$

$$0.9 R_1 = R_1(1 - e^{-\frac{t}{0.596}})$$

$$e^{-\frac{t}{0.596}} = 0.1; \ t = 1.372 \, min$$

thus

$$\frac{R_1}{R_2} = 0.5$$

$$\tau_1 = \tau_2 = 0.596 \ min$$

$$t_{90\%} = 1.372 \ min$$

Comment :
Small values of τ_1, τ_2 *indicate the system regains the steady state quickly. Also as* $R_1 > R_2$,*the* $second$ *ta*
responds more slowly to changes than first $\tan k$.

8.4 Assuming the flow in the   manometer to be laminar function between applied pressure P_1 and the manometer reading h. Calculate a) steady sate gain ,b) τ ,c) ξ. Comment on the parameters and their relation   to the physical nature of this problem.
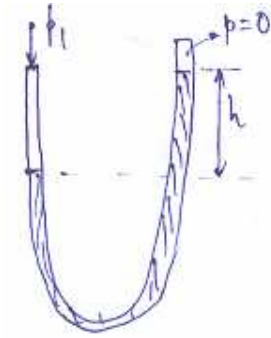
Assumptions:

Cross-sectional area =a
Length of mercury in column = L
Friction factor = 16/Re (laminar flow)
Mass of mercury = m_{rg}

Writing a force balance on the mercury

Mass X acceleration = pressure force - drag force - gravitational force

$$(AL\rho)\frac{d^2h}{dt^2} = Ap_1 - Af\frac{\rho u^2}{2} - A(\rho gh)$$

$$\frac{L}{g}\frac{d^2h}{dt^2} + \frac{8\mu}{\rho gD}\frac{dh}{dt} + h = \frac{p_1}{\rho g}$$

At Steady state, $\quad h_s = \frac{p_{1s}}{\rho g}$

$$= \frac{L}{g}\frac{d^2H}{dt^2} + \frac{8\mu}{\rho gD}\frac{dH}{dt} + H = \frac{p_1}{\rho g}$$

$$= \frac{L}{g}\left[s^2 H(s)\right] + \frac{8\mu}{\rho gD}sH(s) + H(s) = \frac{p_1(s)}{\rho g}$$

$$= \left[k_1 s^2 + k_2 s + 1\right]H(s) = k_3 p_1(s)$$

$$= \frac{H_1(s)}{p_1(s)} = \frac{k_3}{(k_1 s^2 + k_2 s + 1)}$$

Where $k_1 = \dfrac{L}{g}$; $k_2 = \dfrac{8\mu}{\rho g D}$; $k_3 = \dfrac{1}{\rho g}$;

Thus $\dfrac{H_1(s)}{p_1(s)} = \dfrac{R}{(\tau^2 s^2 + 2\xi\tau\ s + 1)}$

Where $R = \dfrac{1}{\rho g}$; $\tau^2 = \dfrac{L}{g}$; $2\xi\tau = \dfrac{8\mu}{\rho g D}$;

Now $b)\, \tau = \sqrt{\dfrac{L}{g}}$;

$$c)\, \xi = \frac{8\mu}{\rho g D} \cdot \frac{1}{2\tau} = \left(\frac{4\mu}{\rho g D}\right)\left(\sqrt{\frac{L}{g}}\right)^{-1}$$

Steady state gain

$$\mathop{Lt}_{S\to 0} G(s) = R = \frac{1}{\rho g};$$

Comment : a) τ is the time period of a simple pendulum of Length L.

         b) ξ is inversely proportional to τ, smaller the τ, the system will tend to move from under damped to over damped characteristics.

8.5 Design a mercury manometer that will measure pressure of upto 2 atm, and give responses that are slightly under damped with $\xi = 0.7$

Parameter to be decide upon :

.a) Length of column of mercury
.b) diameter of tube.
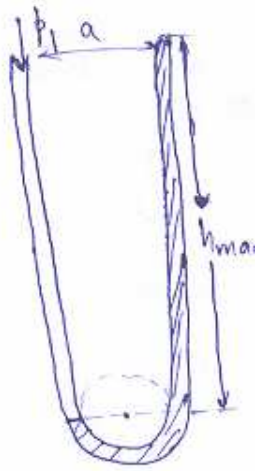
Considering h_{max} to be the maximum height difference to be used

$$p_1 = \rho g h_{max} = h_{max} = \frac{2*1.01325*10^5}{9.81*13600};$$

$$h_{max} = 1.51 m;$$

Assuming the separation between the tubes to be 30 cm,

We get an additional length of 0.47 m;

Which gives us the total length L= 1.5176.47

L = 2 M

Now, $\xi = 0.7 = \left(\frac{4\mu}{\rho g D}\right)\left(\sqrt{\frac{g}{L}}\right) = 0.7$

$$D = \frac{4\mu\sqrt{\frac{g}{L}}}{0.7\rho g} = \frac{4*1.6*10^{-3}*\sqrt{\frac{9.81}{2}}}{0.74*13600*9.81} = 1.5*10^{-7}$$
$$= 0.00015$$

As can be seen, the values yielded are not proper, with too small a diameter and too large a length. A smaller ξ value and lower measuring range of pressure might be better.

8.6 verify that for a second order system subjected to a step response,

$$Y(t) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\frac{\xi t}{\tau}} \sin\left[\sqrt{1-\xi^2}\right]\frac{t}{\tau} + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

With $\xi < 1$

$$Y(s) = \frac{1}{s}\frac{1}{(\tau^2 s^2 + 2\xi\tau s + 1)}$$

$$\tau^2 s^2 + 2\xi\tau s + 1 = (s - s_1)(s - s_2)$$

$$\text{where } s_1 = \frac{-\xi}{\tau} + \frac{\sqrt{\xi^2 - 1}}{\tau} = -a + b$$

$$s_2 = \frac{-\xi}{\tau} + \frac{\sqrt{\xi^2 - 1}}{\tau} = -a - b$$

$$Y(s) = \frac{\left(\frac{1}{\tau^2}\right)}{s(s - s_1)(s - s_2)}$$

$$Y(s) = \frac{1}{\tau^2}\left[\frac{A}{s} + \frac{B}{(s - s_1)} + \frac{C}{(s - s_2)}\right]$$

$$Y(s) = \frac{1}{\tau^2}\left[\frac{A}{s} + \frac{B}{(s - s_1)} + \frac{C}{(s - s_2)}\right]$$

$$A(s^2 - (s_1 + s_s)s + s_1 s_2) + Bs(s - s_2) + Cs(s - s_1) = 1$$

$$A + B + C = 0$$

$$-A(s_1 + s_s) - Bs_2 - Cs_1 = 1$$

$$As_1 s_s = 1; A = \frac{1}{s_1 s_2} = B + C = -\frac{1}{s_1 s_2}$$

$$= Bs_2 + Cs_2 = -\frac{1}{s_1}$$

$$= Cs_1 + Cs_2 = \frac{s_1 + s_2}{s_1 s_2} - \frac{1}{s_1} = \frac{1}{s_2}$$

$$= C = \frac{1}{s_2(s_2 - s_1)} = B = -\frac{1}{s_2(s_2 - s_1)} - \frac{1}{s_1 s_2} = -\frac{1}{s_1(s_2 - s_1)}$$

$$Y(s) = \frac{1}{\tau^2} \left[\left(\frac{1}{s_1 s_2} \right) \cdot \frac{1}{s} - \frac{1}{s_1(s_2 = s_1)} \cdot \frac{1}{(s - s_1)} + \frac{1}{s_2(s_2 - s_1)} \cdot \frac{1}{(s - s_2)} \right]$$

8.6 $$Y(t) = \frac{1}{\tau^2} \left[\frac{1}{s_1 s_2} - \frac{1}{s_1(s_2 - s_1)} e^{S_1 t} + \frac{1}{s_2(s_2 - s_1)} e^{S_2 t} \right]$$

$$\frac{1}{\tau^2 s_1 s_2} = 1$$

$$Y(t) = 1 - \frac{1}{\tau^2(s_2 - s_1)} \left[\frac{1}{s_1} e^{S_1 t} - \frac{1}{s_2} e^{S_2 t} \right]$$

$$Y(t) = 1 - \frac{1}{(s_2 - s_1)} \left[s_2 e^{S_1 t} - s_1 e^{S_2 t} \right]$$

$$Y(t) = 1 + \frac{\tau}{2\sqrt{\xi^2 - 1}} \left[s_2 e^{S_1 t} - s_1 e^{S_2 t} \right]$$

$$Y(t) = 1 - \frac{\bar{g}e^{-\frac{\xi t}{\tau}}}{2\sqrt{\xi^2 - 1}}\left[(-a - jb)(\cos bt + j\sin bt) - (-a + jb)(\cos bt - j\sin bt)\right]$$

$$Y(t) = 1 - \frac{\bar{g}e^{-\frac{\xi t}{\tau}}}{2\sqrt{\xi^2 - 1}}\left[-2jb(b\cos bt + a\sin bt)\right]$$

$$Y(t) = 1 - \frac{e^{-\frac{\xi t}{\tau}}}{\sqrt{\xi^2 - 1}}\left[\sqrt{1 - \xi^2}\cos(\alpha t) + \xi\sin(\alpha t)\right]$$

$$\alpha = \frac{\left[\sqrt{1 - \xi^2}\right]}{\xi}$$

$$\phi = \tan^{-1}\left[\frac{\left[\sqrt{1 - \xi^2}\right]}{\xi}\right]$$

verified

8.14 From the figure in your text Y(4) for the system response is expressed

b) verify that for $\xi = 1$, and a step input

$$Y(t) = 1 - \left(1 + \frac{t}{\tau}\right)e^{-\frac{t}{\tau}}$$

$$Y(s) = \frac{1}{s} \frac{1}{\tau^2 s^2 + \tau s + 1}$$

$$Y(s) \frac{1}{s(\tau s + 1)^2} = \frac{A}{B} + \frac{Bs + C}{(\tau + 1)^2}$$

$$A(\tau^2 s^2 + 2\tau s + 1) + Bs^2 Cs = 1$$

$$A\tau^2 + B = 0$$

$$2A\tau + C = 0$$

A=1; $B = \tau^2$; $C = 2\tau$

$$Y(s) = \frac{1}{s} - \frac{\tau(\tau s + 1) + \tau}{\tau(s + 1)^2}$$

$$Y(s) = \frac{1}{s} - \frac{\tau}{(\tau s + 1)} - \frac{\tau}{(\tau s + 1)^2}$$

$$Y(t) = 1 - e^{-\frac{t}{\tau}} - \frac{1}{\tau} te^{-\frac{t}{\tau}}$$

$$Y(t) = 1 - (1 + \frac{t}{\tau})te^{-\frac{t}{\tau}}$$

proved

c) for $\xi > 1$, prove that the step response is

$$Y(t) = 1 - e^{-\frac{\xi t}{\tau}} \left[\cosh(\alpha t) + \beta \sinh(\alpha t)\right]$$

$$\alpha = \frac{\sqrt{\xi^2 - 1}}{\tau} \quad \beta = \frac{\xi}{\sqrt{\xi^2 - 1}}$$

Now $Y(s) = \dfrac{1/\tau^2}{s(s - B_1)(s - s_2)}$

Where

$$s_1 = -\frac{\xi}{\tau} + \frac{\sqrt{\xi^2 - 1}}{\tau}$$

$$s_2 = -\frac{\xi}{\tau} - \frac{\sqrt{\xi^2 - 1}}{\tau}$$

from 8.6(a)

$$Y(t) = \frac{1}{\tau^2}\left[\frac{1}{s_1 s_2}\frac{1}{s} - \frac{1}{s_1(s_2 - s_1)}\frac{1}{(s - s_1)} + \frac{1}{s_2(s_2 - s_1)}\frac{1}{(s - s_2)}\right]$$

$$Y(t) = 1 - \frac{1}{(s_2 - s_1)}\left[s_2 e^{s_1 t} - s_1 e^{s_2 t}\right]$$

$$Y(t) = 1 + \frac{\tau}{2\sqrt{\xi^2 - 1}}\left[\left(\frac{-\xi - \sqrt{1 - \xi^2}}{\tau}\right)_1 e^{-\frac{\xi}{\tau}}e^{\left(\frac{\sqrt{\xi^2 - 1}}{\tau}\right)t} - \left(\frac{-\xi + \sqrt{1 - \xi^2}}{\tau}\right)e^{-\frac{\xi}{\tau}}e^{-\frac{\sqrt{\xi^2 - 1}}{\tau}t}\right]$$

$$Y(t) = 1 + \frac{e^{-\frac{\xi t}{\tau}}}{2\sqrt{\xi^2 - 1}}\left[-\xi e^{\left(\frac{\sqrt{\xi^2 - 1}}{\tau}\right)t} + \xi e^{-\frac{\xi t}{\tau}} - \sqrt{\xi^2 - 1}e^{-\frac{\sqrt{\xi^2 - 1}\ t}{\tau}} - \sqrt{\xi^2 - 1}e^{-\frac{\sqrt{\xi^2 - 1}}{\tau}t}\right]$$

$$Y(t) = 1 + e^{-\frac{\xi t}{\tau}}\left[-\frac{\xi}{\sqrt{\xi^2 - 1}}\left(\frac{e^{\alpha t} - e^{-\alpha t}}{2}\right) - \left(\frac{e^{\alpha t} + e^{-\alpha t}}{2}\right)\right]$$

$$Y(t) = 1 - e^{-\frac{\xi t}{\tau}}\left[\cosh(\alpha t) + \beta \sinh(\alpha t)\right]$$

8.7 Verify that for a unit step-input

(1) overshoot $= \exp\left(\dfrac{-\pi\xi}{\sqrt{1 - \xi^2}}\right)$

(2) Decay ratio $= \exp\left(\dfrac{-2\pi\xi}{\sqrt{1 - \xi^2}}\right)$

For a unit step input the response (ξ<1):

$$Y(t) = 1 - \frac{e^{-\left(\frac{\xi t}{\tau}\right)}}{\sqrt{1 - \xi^2}} Sin\left(\sqrt{1 - \xi^2}\,\frac{t}{\tau} + \tan^{-1}\left(\frac{\sqrt{1 - \xi^2}}{\xi}\right)\right)$$

(1) we have to find time t where the maxima occurs

$= dY/dt = 0$

$$\frac{dY)}{dt} = \frac{\xi e^{-\left(\frac{\xi t}{\tau}\right)}}{\tau\sqrt{1 - \xi^2}} Sin\left(\sqrt{1 - \xi^2}\,\frac{t}{\tau} + \tan^{-1}\left(\frac{\sqrt{1 - \xi^2}}{\xi}\right)\right)$$

$$-\frac{e^{-\left(\frac{\xi t}{\tau}\right)}}{\tau} Cos\left(\sqrt{1 - \xi^2}\,\frac{t}{\tau} + \tan^{-1}\left(\frac{\sqrt{1 - \xi^2}}{\xi}\right)\right) = 0$$

$$= \tan\left(\sqrt{1 - \xi^2}\,\frac{t}{\tau} + \tan^{-1}\left(\frac{\sqrt{1 - \xi^2}}{\xi}\right)\right) = \frac{\sqrt{1 - \xi^2}}{\xi}$$

$$\frac{\sqrt{1 - \xi^2}\,t}{\xi} = n\pi$$

for maxima

$$= \frac{\sqrt{1 - \xi^2}\,t}{\xi} = 2n\pi$$

$$= \quad t = \frac{\pi t}{\sqrt{1 - \xi^2}}$$

8.8 Verify that for X(t) =A sin ωt, for a second order system,

$$Y(t) = \frac{A}{\sqrt{\left(1 - (\omega t)^2\right)^2 + (2\xi\tau)^2}} \sin(\omega t + \phi)$$

$$\phi = -\tan^{-1} \frac{2\xi\omega\tau}{1 - (\omega\tau)^2}$$

$$Y(s) = \frac{A}{(s^2 + \omega^2)} \frac{1}{(\tau^2 s^2 + 2\xi\tau s + 1)}$$

$$Y(s) = \frac{A\omega}{\tau^2} \left[\frac{A^1}{(s - j\omega)} + \frac{B^1}{s + j\omega} + \frac{C^1}{(s - s_1)} + \frac{D^1}{(s - s_2)} \right]$$

Now as $t \to \infty, Y(t) = A^{11} \cos \omega t + B^{11} \sin \omega t$

Where $A^{11} = A^1 + B^1$

$B^{11} = j(A^1 - B^1)$

to determine A^1, B^1 put $s = j\omega, -j\omega$ in the order

$$A^1 = \frac{-j}{2\omega(j\omega - s_1)(j\omega - s_2)} \qquad B^1 = \frac{j}{2\omega(j\omega + s_1)(j\omega + s_2)}$$

$$A^{11} = \frac{j}{2\omega} \left[\frac{1}{(s_1 + j\omega)(s_2 + j\omega)} - \frac{1}{(j\omega - s_1)(j\omega - s_2)} \right]$$

$$A^{11} = \frac{j}{2\omega}\left[\frac{(-\omega^2 - js_1\omega - js_2\omega + s_1s_2) - (-\omega^2 + js_1\omega + js_2\omega + s_1s_2)}{(s_1^2 + \omega^2)(s_2^2 + \omega^2)}\right]$$

$$A^{11} = \left[\frac{(s_1 + s_2)}{(s_1^2 + \omega^2)(s_2^2 + \omega^2)}\right] \quad \text{similarly} \quad B^{11} = \left[\frac{(s_1s_2 - \omega^2)}{\omega(s_1^2 + \omega^2)(s_2^2 + \omega^2)}\right]$$

using $\quad s_1 + s_2 = \dfrac{-2\xi}{\tau} \quad s_1s_2 = \dfrac{1}{\tau^2}$

$$= s_1^2 + s_2^2 = \frac{4\xi^2}{\tau^2} - \frac{2}{\tau^2} = \frac{2(2\xi^2 - 1)}{\tau^2}$$

$$A^{11} = \frac{A\omega}{\tau^2}\left[\frac{-\dfrac{2\xi}{\tau}}{\dfrac{1}{\tau^4} + \dfrac{2\omega^2}{\tau^2}(2\xi^2 - 1) + \omega^4}\right]$$

$$= \frac{\dfrac{-2A\omega\xi}{\tau^3}}{\left(\omega^2 - \dfrac{1}{\tau^2}\right)^2 + \left(\dfrac{2\xi\omega}{\tau}\right)^2} = \frac{-2A\omega\xi\tau}{(1 - (\omega\tau)^2)^2 + (2\xi\omega\tau)^2}$$

and $\quad B^{11} = \dfrac{A\varpi}{\tau}\left[\dfrac{\left(\dfrac{1}{\tau^2} - \omega^2\right)}{\omega\left(\left(\dfrac{1}{\tau^2} - \omega^2\right)^2 + \left(\dfrac{2\xi\omega}{\tau}\right)^2\right)}\right]$

$$= \frac{A(1 - (\omega\tau)^2)}{(1 - (\omega\tau)^2)^2 + (2\xi\omega\tau)^2}$$

Thus $\tan\phi = \dfrac{A^{11}}{B^{11}} = \dfrac{-2\omega\tau\xi}{1-(\omega\tau)^2}$

And, $A_{New} = \dfrac{A}{\sqrt{((1-(\omega\tau)^2)^2 + (2\xi\omega\upsilon)^2}}$ (using $\sqrt{A^{11^2} + B^{11^2}} = A_{New}$

Thus, $Y(t) = \dfrac{A}{\sqrt{((1-(\omega\tau)^2)^2 + (2\xi\omega\upsilon)^2}} Sin(\omega t + \phi)$

proved

8.9) If a second- order system is over damped, it is more difficult to determine the parameters $\xi \,\&\, \tau$ experimentally. One method for determining the parameters from a step response has been suggested by R.c Olderboung and H.Sartarius (The dynamics of Automatic controls,ASME,P7.8,1948),as described below.

(a) Show that the unit step response for the over damped case may be written in the form.

$$s(t) = 1 - \dfrac{r_1 e^{r_2 t} - r_2 e^{r_1 t}}{r_1 r_2}$$

Where r_1 and r_2 are the roots of

$\tau^2 s^2 + 2\xi\tau s + 1 = 0$

(b) Show that s(t) has an inflection point at

$$t_i = \dfrac{\ln(r_2/r_1)}{(r_2 - r_1)}$$

© Show that the slope of the step response at the inflection point

$$\dfrac{d(s)}{dt}\Big|_{t-t_i} = s^1(t_i)$$

Where, $s^1(t_i) = -r_1 e^{r_1 t} = -r_2 e^{r_2 t_i}$

$$=-r_1\left(\frac{r_2}{r_1}\right)^{r_1(r_1-r_2)}$$

(d) Show that the value of step response at the inflection point is

$$s^1(t_i)=1+\frac{r_1r_2}{r_1r_2}s^1(t_i)\text{ and that hence }1-\frac{s(t_i)}{s^1(t_i)}=-\frac{1}{r_1}-\frac{1}{r_2}$$

(e) on a typical sketch of a unit step response show distances equal to

$$1-\frac{s(t_i)}{s^1(t_i)}\ \&\ \frac{1}{s^1(t_i)}$$

(f) Relate $\xi\ \&\ \tau$ to $r_1\ \&\ r_2$

(a) $G(s)=\dfrac{1}{\tau^2 s^2+2\xi\tau s+1)}=\dfrac{\dfrac{1}{\tau^2}}{\left(s^2+\left(\dfrac{2\xi}{\tau}\right)s+\dfrac{1}{\tau^2}\right)}=\dfrac{\dfrac{1}{\tau^2}}{(s-r_1)(s-r_2)}$

$$=\ Y(s)=\frac{\dfrac{1}{\tau^2}}{s(s-r_1)(s-r_2)}$$

$$\frac{1}{s(s-r_1)(s-r_2)}=\frac{A}{s}+\frac{B}{s-r_1}+\frac{C}{s-r_2}$$
$$1=A(s-r_1)(s-r_2)+Bs(s-r_2)+cs(s-r_1)$$

Put s= 0 = Ar₁r₂ =1 ; $A=\tau^2$

Put $s=r_1=Br_1(r_1-r_2)=1;B=\dfrac{1}{r_1(r_1-r_2)}$

$s=r_2=Cr_2(r_2-r_1)=1;C=\dfrac{1}{r_2(r_2-r_1)}$

$$Y(s)=\frac{1}{\tau^2}\left(\frac{\tau^2}{s}+\frac{1}{r_1(r_1-r_2)(s-r_1)}+\frac{1}{r_2(r_2-r_1)(s-r_2)}\right)$$

$$Y(t) = \frac{1}{\tau^2}\left(\tau^2 + \frac{e^{r_1 t}}{r_1(r_1 - r_2)} + \frac{e^{r_2 t}}{r_2(r_2 - r_1)}\right)$$

$$Y(t) = 1 - \left(\frac{1}{r_1 - r_2}\left[r_1 e^{r_2 t} - r_2 e^{r_1 t}\right]\right)$$

$$Y(t) = 1 - \frac{\phi r_1 e^{r_2 t} - r_2 e^{r_1 t}}{(r_1 - r_2)}$$

(b) For inflection point, $\dfrac{d^2 s}{dt^2} = 0$ & $\dfrac{d^3 s}{dt^2} = 0$

$$\frac{ds}{dt} = -\frac{r_1 r_2 (e^{r_2 t} - e^{r_2 t})}{r_1 - r_2}$$

$$\frac{d^2 s}{dt^2} = -\frac{r_1 r_2 (r_2 e^{r_2 t} - r_1 e^{r_2 t})}{r_1 - r_2} = 0$$

$$= u_2 e^{r_2 t} = r_1 e^{r_1 t} = \frac{r_2}{r_1} = e^{(r_1 - r_2) t_i}$$

$$= t_i = \frac{\ln\left(\dfrac{r_2}{r_1}\right)}{r_1 - r_2}$$

(c) $\dfrac{ds(t)}{dt}\bigg|_{t = t_i} = s'(t_i)\,]$

$$= -\frac{r_1 r_2}{r_1 - r_2}\left[\left(\frac{r_2}{r_1}\right)^{\frac{r}{r_1 - r_2}} - \left(\frac{r_2}{r_1}\right)^{\frac{r_1}{r_2 - r_1}}\right]$$

$$= -\frac{r_1 r_2}{r_1 - r_2}\left[\left(\frac{r_2}{r_1}\right)^{\frac{r}{r_1-r_2}} - \frac{r_1^2}{r_{1-}r_2}\left(\frac{r_2}{r_1}\right)^{\frac{r_1}{r_2-r_1}}\right]$$

$$= -\frac{r_1(r_1 - r_2)}{r_1 - r_2}\left[\left(\frac{r_2}{r_1}\right)^{\frac{r_1}{r_2-r_1}}\right]$$

$$\frac{ds(t)}{dt}\Big|_{t=t_i} = -r_1\left[\left(\frac{r_2}{r_1}\right)^{\frac{r_1}{r_2-r_1}}\right]$$

Also $\dfrac{ds(t)}{dt}\Big|_{t=t_i} = -\dfrac{r_1 r_2 (e^{r_2 t} - e^{r_1 t})}{(r_1 - r_2)}$

$$\frac{ds(t)}{dt}\Big|_{t=t_i} = -\frac{r_1 r_2 - e^{r_1 t}}{(r_1 - r_2)}\left(\frac{r_1}{r_2} - 1\right)$$

$$\frac{ds(t)}{dt}\Big|_{t=t_i} = -r_1 e^{r_1 t} = -r_2 e^{r_2 t}$$

(d) $s(t_i) = 1 - \dfrac{r_1 e^{r_2 t_i} - r_2 e^{r t_{i1}}}{r_1 - r_2} = 1 + \dfrac{s^1(t_i)\left[\dfrac{r_1}{r_2} - \dfrac{r_2}{r_1}\right]}{r_1 - r_2}$

$$= \quad s(t_i) = = 1 + \frac{s^1(t_i)\left[\dfrac{r_1}{r_2} - \dfrac{r_2}{r_1}\right]}{r_1 - r_2}$$

Now

$$s(t_i) - 1 = = s^1(t_i)\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

$$\frac{1 - s(t_i)}{s^{'}(t)} = = \left[-\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$r_1 + r_2 = \frac{1}{\tau^2} = \sqrt{r_1 r_2} = \frac{1}{\tau}; \tau = \frac{1}{\sqrt{r_1 r_2}}$$

$$r_1 + r_2 = \frac{-2\xi}{\tau} =; \ r_1 + r_2 = -2\sqrt{r_1 r_2}\xi$$

$$\xi = -\frac{1}{2}\left[\sqrt{\frac{r_1}{r_2}} + \sqrt{\frac{r_2}{r_1}}\right]$$

proved.

8.10 Y(0),Y(0.6),Y(∞) if

$$Y(s) = \frac{1}{s}\frac{25(s+1)}{(s^2 + 2s + 1)}$$

$$Y(s) = \left(1 + \frac{1}{s}\right)\frac{1}{(\frac{s^2}{25} + \frac{2s}{25} + 1)}$$

Y(s) impulse response + step response of G(s)

Where $G(s) = \dfrac{1}{(\dfrac{s^2}{25} + \dfrac{2s}{25} + 1)}$

$$Y(t) = \frac{1}{\sqrt{1-\xi^2}\,\tau}\, e^{-\frac{\xi t}{\tau}} \sin\sqrt{1-\xi^2}\,\frac{t}{\tau} + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

Y(t) = 1+5.0.3e^{-t} sin (4.899t)-1.02e^{-t} sin(4.899t+1.369)

Y(0)= 1-1=0
Y(0.6) = 1+0.561+0.515
Y(∞) =1

Comment : as we can see ,the system exhibits an inverse response by increasing from zero to more than 1 and as t tend to ∞,will reach the steady state value of 1.

8.11 In the system shown the dev in flow to tank 1 is an impulse of magnitude 5 . A$_1$ = 1 ft^2, A$_2$ = A$_3$ = 2 ft^2 , R$_1$ = 1 ft/cfm R$_2$ = 1.5 ft/cfm .

(a) Determine H$_1$(s), H$_2$(s), H$_3$(s)
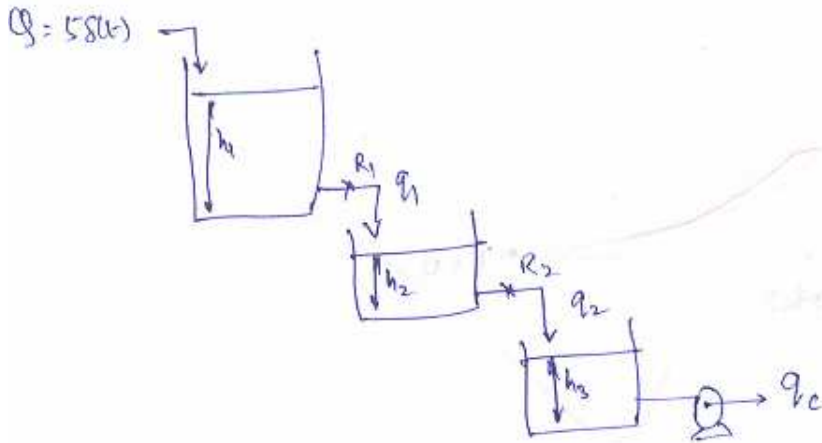
Transfer function for tank 1 $\dfrac{H_1(s)}{Q(s)} = \dfrac{1}{(\tau_1 s + 1)}$

$$H_1(s) = \frac{5}{(s+1)}$$

from tank 2, $\dfrac{H_2(s)}{Q(s)} = \dfrac{R_2}{(\tau_1 s + 1)(\tau_2 s + 1)} = \dfrac{1.5}{(s+1)(3s+1)}$

for tank 3, $q_2 - q_c = A_3 \dfrac{dh_3}{dt}$

$q_3 = q_c (const) = Q_2 = A_3 \dfrac{dh_3}{dt}$

$\dfrac{H_2}{R_2} = A_3 \dfrac{dh_3}{dt}$

thus, $A_3 SH_3(s) = \dfrac{H_2(s)}{R_2} = \dfrac{H_3(s)}{H_2(s)} = \dfrac{1}{3s}$

$A_3 SH_3(s) = \dfrac{H_2(s)}{R_2} = \dfrac{H_3(s)}{H_2(s)} = \dfrac{1}{3s}$

$\dfrac{H_3(s)}{Q(s)} = \dfrac{0.5}{s(s+1)(3s+1)}$

8.11© $H_1(s) = \dfrac{5}{(s+1)}$

$H_1(t) = 5e^{-t}$

$H_1(3.46) = 0.155A$

$\dfrac{H_2(s)}{Q(s)} = \dfrac{1.5}{3s^2 + 4s + 1}$

$Q(1) = 5 = H_2(s) = \dfrac{7.5}{3s^2 + 4s + 1}$

$$\tau = \sqrt{3}$$

$$2\xi\tau = 4$$

$$\xi = \frac{4}{2\tau} = \frac{4}{2\sqrt{3}} = 1.155$$

from   fig 8.5

$$\xi = 1.155 \ and \ \frac{t}{\tau} = \ \xi = \frac{t}{\tau} = \frac{3.46}{\sqrt{3}} = 2$$

$$\tau H_2(t) = 0.265 X 7.5$$

$$H_2(t) = \frac{0.265 X 7.5}{\tau} = 1.147$$

$$\frac{H_3(s)}{Q(s)} = \frac{0.5}{s(3s^2 + 4s + 1)}$$

$$Q(s) = 5 = H_3(s) = \frac{2.5}{s(3s^2 + 4s + 1)}$$

$$\tau = \sqrt{3} \ \xi = \frac{2}{\sqrt{3}}$$

from fig 8.2   at $\frac{t}{\tau} = 2, \xi = 1.155$

Y(t) =0.54

H_3(t) =0.54*2.5 = 1.35

8.12 sketch the response Y(t) if $Y(s) = \dfrac{e^{-2S}}{(s^2 + 1.2s + 1)}$

Determine Y(t) for t = 0,1,5,∞

$$Y(s) = \frac{e^{-2S}}{(s^2 + 1.2s + 1)} = \frac{e^{-2S}}{(s+0.6)^2 + (0.8)^2} = \frac{1}{0.8}\frac{e^{-2S}(0.8)}{(s+0.6)^2 + (0.8)^2}$$

$$Y(t) = 1.25e^{-.6(t-2)}\sin(0.8(t-2))\ t \geq 2$$

for $t = 0\ Y(0) = 0$

$t = 1, Y(1) = 0$

$t = 5, Y(5) = 0.14$

$t = \infty, Y(\infty) = 0$
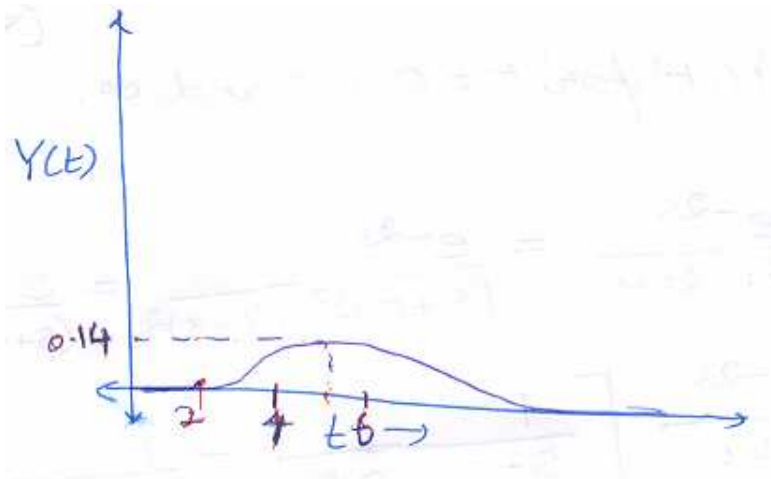
Problem 8.13 The system shown is at steady state at t = 0, with   q = 10 cfm

$A_1 = 1ft^2, A_2 = 1.25ft^2,\ R_1 = 1$ **ft/cfm,** $R_2 = 0.8$ **ft/cfm.**

a) **If flow changes fro 10 to 11 cfm, find** $H_2(s)$.
b) **Determine** $H_2(1), H_2(4), H_2(\infty)$
c) **Determine the initial levels** $h_1(0), h_2(0)$ **in the tanks.**
d) **obtain an expression for** $H_1(s)$ **for unit step change.**
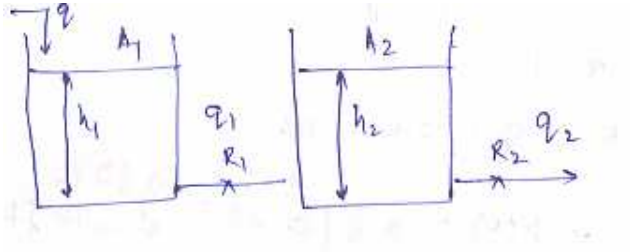
Writing mass balances,

$$q - \frac{(h_1 - h_2)}{R_1} = A_1 \frac{dh_1}{dt} \ (for \tan k \ 1)$$

At steady state $q_2 - \frac{(h_{1s} - h_{2s})}{R_1} = h_{1S} - h_{2S}$

Also for tank 2
$$\frac{(h_1 - h_2)}{R_1} - \frac{h_2}{R_2} = A_2 \frac{dh_2}{dt}$$

At steady state $\frac{(h_{1s} - h_{2s})}{1} = \frac{h_{2S}}{0.8} = h_{2S} = 0.8 * 10 = 8$

$h_{1S} = 18$

C) $h_{1S} = 18 \, ft \ h_2 \, (0) = 8 \, ft$

The equations in terms of deviation variables

$$Q - Q_1 = A_1 \frac{dH_1}{dt} \quad \text{where} \quad Q_1 = \frac{H_1 - H_2}{R_1}$$

$$Q_1 - Q_2 = A_2 \frac{dH_2}{dt} \quad Q_2 = \frac{H_2}{R_2}$$

$$\frac{H_2(s)}{Q(s)} = \frac{R_2}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2 + A_1 R_2)s + 1} = \frac{0.8}{s^2 + 2.8s + 1}$$

$$H_2(s) = \frac{0.8}{s(s^2 + 2.8s + 1)} \quad (Ans\, 8.31(a))$$

Step response of a second order system

$$\tau^2 = 1 = \tau = 1$$

$$2\xi\tau = 2.8; \xi = \frac{2.8}{2} = 1.4$$

$a)t = 1 = \dfrac{t}{\tau} = 1; H_2(t) = 0.8(0.22) = 0.176\, ft(\,from\, fig)$

$b)t = 4 = \dfrac{t}{\tau} = 4; H_2(t) = 0.8(0.78) = 0.624\, ft(\,from\, fig)$

$c)t \to \infty = H_2(t \to \infty) = 0.8\, ft$

Thus
$H_2(1) = 0.176\, ft$
$H_2(4) = 0.624\, ft$
$H_2(\infty) = 0.8\, ft$

8.13(d) we have

$$Q(s)Q_1(s) = A_1(s)H_1(s)$$

$$Q_1(s) - Q_2(s) = A_2(s)H_2(s)$$

$$Q(s) - Q_2(s) = A_1(s)H_1(s) + A_2(s)H_2(s)$$

$$Q(s) - A_1(s)H_1(s) = \left(\frac{1}{R_2} + A_2(s)H_2(s)\right)$$

$$= \left(\left(\frac{1 + \tau_2 s}{R_2} \right) H_2(s) \right)$$

$$H_2(s) = \left(\frac{R_2 (Q(s) - A_1 s H_1(s))}{H \tau_2 s} \right)$$

We have $\dfrac{H_2(s)}{Q(s)} = \left(\dfrac{R_2}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2 + A_1 R_2)s + 1} \right) = \dfrac{R_2}{Deg}$

$$\frac{R_2 H_2(s)}{Deg} = \left(\frac{R_2 (Q(s) - A_1 s H_1(s))}{(1 + \tau_2 s)} \right)$$

$$\frac{1 + \tau_2 s}{Deg} = \left(1 - A_1 s - \frac{H_1(s)}{Q(s)} \right)$$

$$\frac{H(s)}{Q(s)} = \frac{1}{sA_1} \left(1 - \frac{H \tau_2(s)}{Deg} \right) = \frac{1}{A_1 s} \left[\frac{Deg - 1 - \tau_2 s}{Deg} \right]$$

$$\frac{H(s)}{Q(s)} = \frac{1}{sA_1} \left(\frac{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2 + A_1 R_2)s + 1 - 1 - \tau_2 s}{Deg} \right)$$

$$\frac{H(s)}{Q(s)} = \frac{1}{sA_1} \left(\frac{\tau_1 \tau_2 s + \tau_1 + A_1 R_2)s}{Deg} \right) =$$

$$\frac{H(s)}{Q(s)} = \left(\frac{R_2 + R_1(1 + \tau_2 s)}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2 + A_1 R_2)s + 1} \right)$$
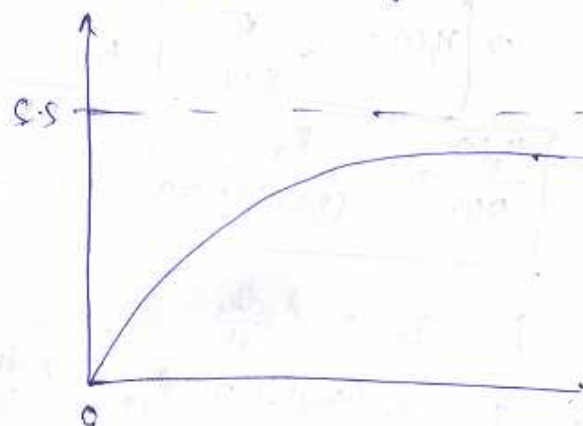
Responses

$H_1(t)$

$H_2(t)$
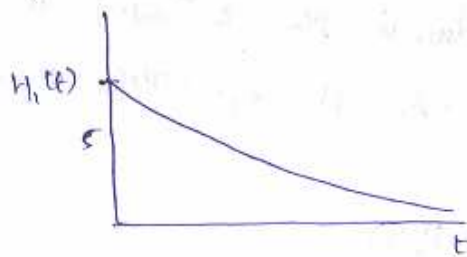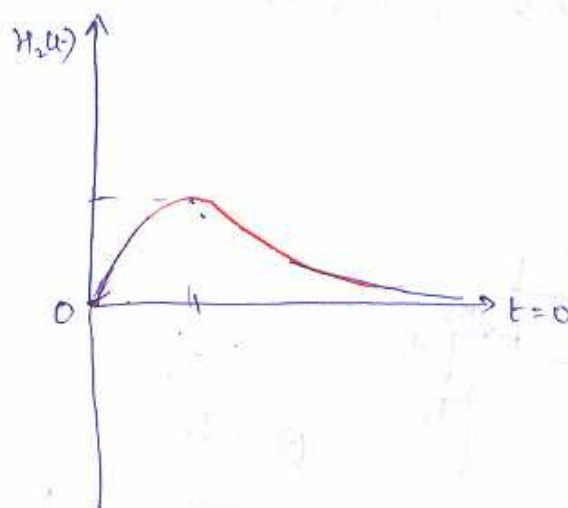
$H_3(t)$ $\zeta = \frac{2}{\sqrt{3}}$ ⟹ System is overdamped.

8.14

$$Y(s) = \frac{2}{s} \frac{(2s+4)}{(4s^2 + 0.8s + 1)}$$

$$Y(s) = \frac{4}{s} \frac{(s+2)}{(4s^2 + 0.8s + 1)}$$

$$Y(s) = 4\left(1 + \frac{2}{s}\right) \frac{1}{(4s^2 + 0.8s + 1)}$$

$$Y(s) = \frac{8}{s(4s^2 + 0.8s + 1)} + \frac{4}{(4s^2 + 0.8s + 1)}$$

$$= \text{(step response)} + \text{(impulse response)}$$

$$Now, \tau = \sqrt{4} = 2 \,;\, 2\xi\tau = 0.8$$

$$\xi = 0.2$$

$$also, \frac{t}{\tau} = \frac{4}{2} = 2$$

impulse response $\tau Y(t) = 4*0.63 = 2.52$ (from figure)

step response $= 8*1.15 = 9.2$ (from figure)

Y(4) $= 1.26+9.2$

Y(4) $=10.46$

Q 9.1. Two tank heating process shown in fig. consist of two identical, well stirred tank in series. A flow of heat can enter tank2. At t = 0 , the flow rate of heat to tank2 suddenly increased according to a step function to 1000 Btu/min. and the temp of inlet Ti drops from $60^{\circ}F$ to $52^{\circ}F$ according to a step function. These changes in heat flow and inlet temp occurs simultaneously.

(a)   Develop a block diagram that relates the outlet temp of tank2 to inlet temp of tank1 and flow rate to tank2.

(b)   Obtain an expression for $T_2'(s)$

(c)   Determine $T_2(2)$ and $T_2(\infty)$

(d)   Sketch the response $T_2'(t)$ Vs t.

Initially Ti = T_1 = T_2 = $60^{\circ}F$ and q=0

W = 250 lb/min

Hold up volume of each tank = 5 ft^3

Density of the fluid = 50 lb/ft^3

Heat Capacity = 1 Btu/lb ($^{\circ}F$)
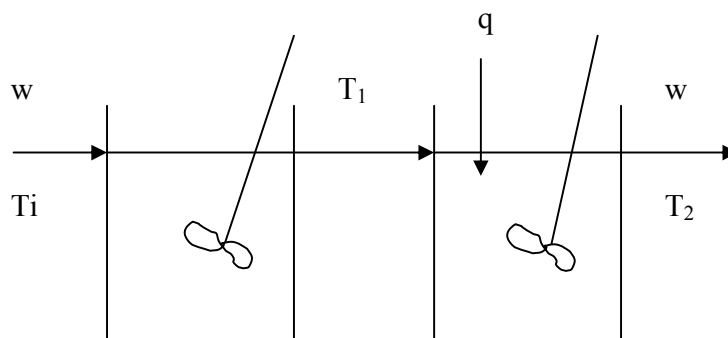
Solution:

(a) For tank 1

Input – output = accumulation

$$WC(T_i - T_o) - WC(T_1 - T_o) = \rho C V \frac{dT_1}{dt} \quad \text{----------------------------- (1)}$$

At steady state

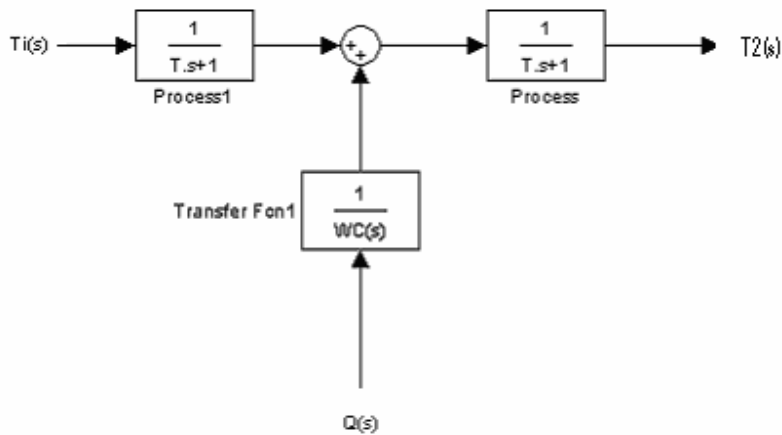$$WC(T_{is} - T_o) - WC(T_{1s} - T_o) = 0 \quad \text{------------------------------------(2)}$$

(1) – (2) gives

$$WC(T_i - T_{is}) - WC(T_1 - T_{1s}) = \rho C V \frac{dT'_1}{dt}$$

$$WT_i' - WT_1' = \rho V \frac{dT'_1}{dt}$$

Taking Laplace transform

$$WT_i(s) = WT_1(s) + \rho V s T_1(s)$$

$$\frac{T_1(s)}{T_i(s)} = \frac{1}{1 + \tau s} \text{, where } \tau = \rho V / W.$$

From tank 2

$$q + WC(T_1 - T_o) - WC(T_2 - T_o) = \rho C V \frac{dT_2}{dt} \quad \text{--------------------------- (3)}$$

At steady state

$$q_s + WC(T_{1s} - T_o) - WC(T_{2s} - T_o) = 0 \quad \text{------------------------------------(4)}$$

(3) – (4) gives

$$Q' + WC(T_1 - T_{1s}) - WC(T_2 - T_{2s}) = \rho C V \frac{dT'_2}{dt}$$

$$Q' + WCT_1' - WCT_2' = \rho C V \frac{dT'_2}{dt}$$

Taking Laplace transform

$Q(s) + WC(T_1(s) - T_2(s)) = \rho C V s T_2(s)$

$$T_2(s) = \frac{1}{1+\tau s}\left[\frac{Q(s)}{WC} + T_1(s)\right], \text{ where } \tau = \rho \ V / W.$$

(b) $\tau = 50*5/250 = 1$ min

WC $= 250*1 = 250$

Ti(s) $= -8/s$ and Q(s) $= 1000/s$

Now by using above two equations we relate T_2 and Ti as below and after taking laplace transform we will get $T_2(t)$

$$T_2(s) = \frac{1}{1+\tau s}\frac{Q(s)}{250} + \frac{1}{(1+\tau s)^2}T_i(s)$$

$$T_2(s) = \frac{4}{(1+s)} - \frac{8}{(1+s)^2}$$

$$T_2(s) = 4\left(\frac{1}{s} - \frac{1}{(1+s)}\right) - 8\left(\frac{1}{s} - \frac{1}{s+1} - \frac{1}{(1+s)^2}\right)$$

$$T_2(t) = (4+8t)e^{-t} - 4$$

(c) $T_2'(2) = -1.29$

$T_2(2) = T_2'(2) + T_{2s} = 60 - 1.29 = 58.71 \ ^oF$

$T_2'(\infty) = -4$

$T_2(\infty) = T_2'(\infty) + T_{2s} = 60 - 4 = 56 \ ^oF$
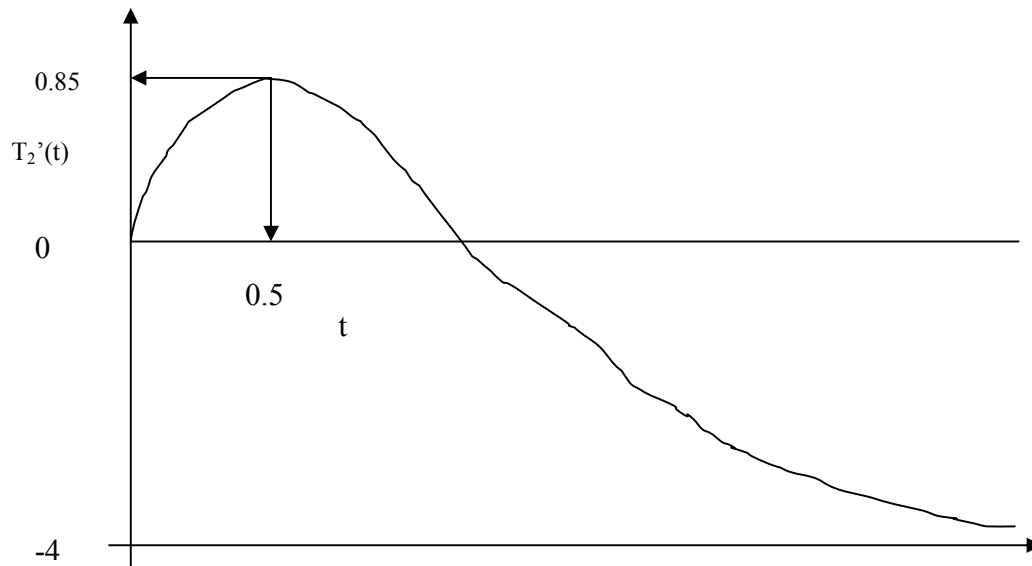
Q – 9.2. The two tank heating process shown in fig. consist of two identical , well stirred tanks in series. At steady state Ta = Tb = 60°F. At t = 0 , temp of each stream changes according to a step function

$Ta'(t) = 10\ u(t)$ $Tb'(t) = 20\ u(t)$

(a) Develop a block diagram that relates T_2' , the deviation in the temp of tank2, to Ta' and Tb'.
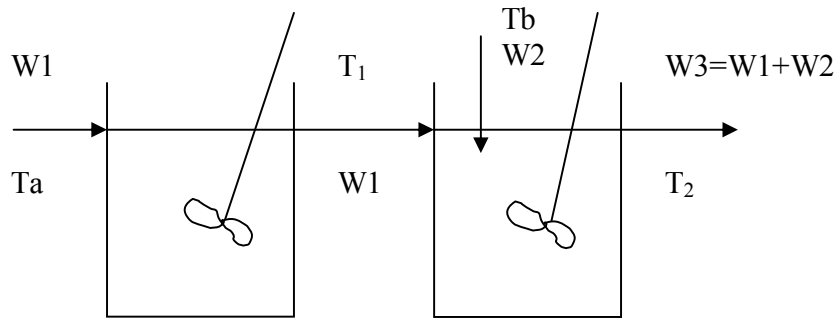
(b) Obtain an expression for $T_2'(s)$

(c) Determine $T_2(2)$

$W1 = W2 = 250\ lb/min$

$V1 = V2 = 10\ ft^3$

$\rho 1 = \rho 2 = 50\ lb/ft^3$

$C = 1\ Btu/lb\ (^oF)$

Solution:

(a)　　For tank1

$$\frac{T_1(s)}{Ta(s)} = \frac{1}{1+\tau_1 s} \text{ , where } \tau_1 = \rho \, V \, / \, W1.$$

　　　For tank2

W1C($T_1 - T_o$) +W2C(Tb $- T_o$) $-$ (W1+W2)C($T_2 - T_o$)= ρ C V $\dfrac{dT_2}{dt}$ ------ (1)

At steady state

W1C($T_{1s} - T_o$) +W2C($T_{bs} - T_o$) $-$ (W1+W2)C($T_{2s} - T_o$)= 0 ----------------(2)

(1) – (2)

W1$T_1{'}$ + W2Tb${'}$ - W3$T_2{'}$ = ρ V $\dfrac{dT{'}_2}{dt}$

Taking L.T

W1T_1(s) + W2Tb(s)- W3T_2(s) = ρVs T_2(s)

$$T_2(s) = \frac{1}{1+\tau s}\left[\frac{W1}{W3}T_1(S) + \frac{W2}{W3}T_b(S)\right] \text{ where } \tau = \rho\ V\ /\ W3.$$
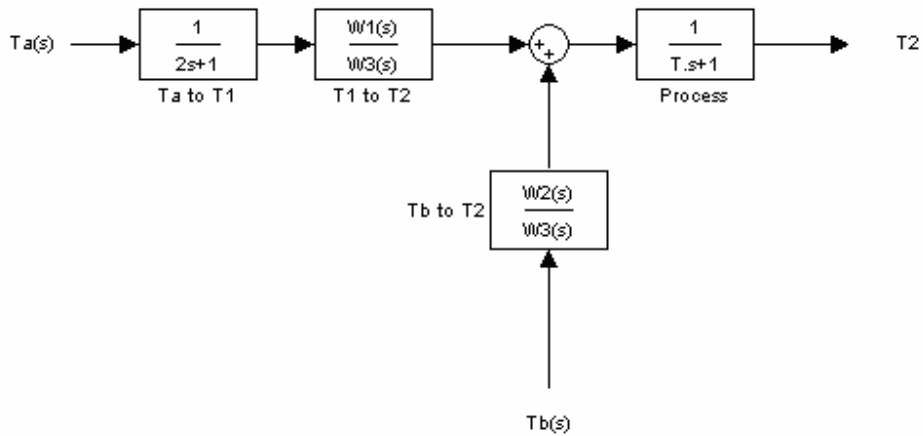
(b) $\tau_1 = 50*10/250 = 2$ min

$\tau = 50*5/250 = 1$ min

W1/W3 = 1/2 = W2/W3

Ta(s) = 10/s and Tb(s) = 0/s

Now by using above two equations we relate T_1 and Ta as below and after taking laplace transform we will get $T_2(t)$

$$T_2(s) = \frac{\frac{1}{2}T_1(s)}{(1+s)} - \frac{\frac{1}{2}T_b(s)}{(1+s)}$$

$$T_2(s) = \frac{\frac{1}{2}T_a(s)}{(1+s)(1+2s)} - \frac{\frac{1}{2}T_b(s)}{(1+s)}$$

$$T_2(s) = \frac{5}{s(1+s)(1+2s)} - \frac{10}{s(1+s)}$$

$$T_2(s) = \frac{15+20s}{s(1+s)(1+2s)}$$

$$T_2(s) = \left(\frac{15}{s} - \frac{5}{s+1} - \frac{20}{(1+2s)}\right)$$

$$T_2(t) = 15 - 5e^{-t} - 10e^{-t/2}$$

(c) $T_2'(2) = 10.64\ ^oF$

   $T_2(2) = T_2'(2) + T_{2s} = 60 + 10.64 = 70.64\ ^oF$

Q – 9.3. Heat transfer equipment shown in fig. consist of tow tanks, one nested inside the other. Heat is transferred by convection through the wall of inner tank.

1. Hold up volume of each tank is 1 ft^3
2. The cross sectional area for heat transfer is 1 ft^2
3. The over all heat transfer coefficient for the flow of heat between the tanks is 10 Btu/(hr)(ft^2)(^{o}F)
4. Heat capacity of fluid in each tank is 2 Btu/(lb)(^{o}F)
5. Density of each fluid is 50 lb/ft^3

Initially the temp of feed stream to the outer tank and the contents of the outer tank are equal to 100 ^{o}F. Contents of inner tank are initially at 100 ^{o}F. the flow of heat to the inner tank (Q) changed according to a step change from 0 to 500 Btu/hr.

(a) Obtain an expression for the laplace transform of the temperature of inner tank T(s).

(b) Invert T(s) and obtain T for t= 0,5,10, ∞
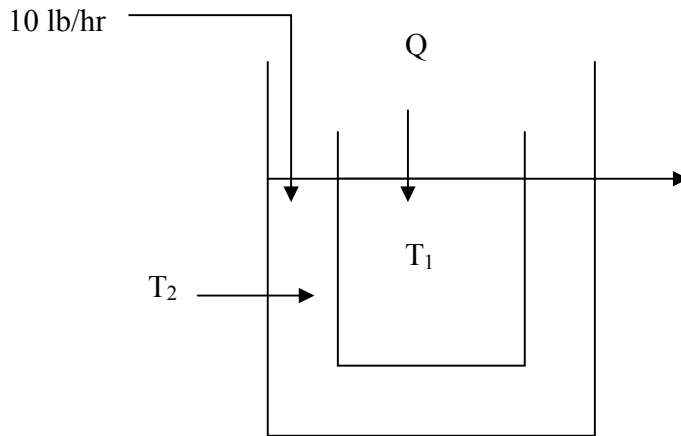
Solution:

    (a)   For outer tank

$$WC(Ti - T_o) + hA(T_1 - T_2) - WC(T_2 - T_o) = \rho C V_2 \frac{dT_2}{dt} \quad\text{----------------------------- (1)}$$

At steady state

$$WC(Tis - T_o) + hA(T_{1s} - T_{2s}) - WC(T_{2s} - T_o) = 0 \quad\text{---------------------------------- (2)}$$

(1) – (2) gives

$$WCTi' + hA(T_1' - T_2') - WCT_2' = \rho C V_2 \frac{dT_2'}{dt}$$

Substituting numerical values

$$10\,Ti' + 10(T_1' - T_2') - 10\,T_2' = 50\frac{dT_2'}{dt}$$

Taking L.T.

$Ti(s) + T_1(s) - 2T_2(s) = 5\,s\,T_2(s)$

Now $Ti(s) = 0$, since there is no change in temp of feed stream to outer tank. Which gives

$$\frac{T_2(s)}{T_1(s)} = \frac{1}{2 + 5s}$$

For inner tank

$$Q - hA\ (T_1 - T_2) = \rho\ C\ V_1\ \frac{dT_1}{dt} \quad \text{--------------------- (3)}$$

$$Qs - hA\ (T_{1s} - T_{2s}) = 0 \quad \text{------------------------------ (4)}$$

(3) – (4) gives

$$Q' - hA\ (T_1' - T_2') = \rho\ C\ V_1\ \frac{dT_1'}{dt}$$

Taking L.T and putting numerical values

$$Q(s) - 10\ T_1(s) + 10\ T_2(s) = 50\ s\ T_1(s)$$

$$Q(s) = 500/s \quad \text{and} \quad T_2(s) = T_1(s)\ /\ (2 + 5s)$$

$$\frac{500}{s} - 10T_1(s) + \frac{10T_1(s)}{2 + 5s} = 50sT_1(s)$$

$$\frac{50}{s} = T_1(s)\left[5s - \frac{1}{2 + 5s} + 1\right]$$

$$T_1(s) = \frac{50(2 + 5s)}{s(25s^2 + 15s + 1)}$$

$$T_1(s) = \frac{2(2 + 5s)}{s\left(s + 3.82\big/50\right)\left(s + 26.18\big/50\right)}$$

$$T_1(s) = \frac{100}{s} - \frac{94.71}{\left(s + 3.82\big/50\right)} - \frac{5.29}{\left(s + 26.18\big/50\right)}$$

$$T_1'(t) = 100 - 94.71\,e^{-3.82t\big/50} - 5.29\,e^{-26.18t\big/50}$$

and

$$T_1(t) = 200 - 94.71\,e^{-3.82t\big/50} - 5.29\,e^{-26.18t\big/50}$$

For t=0,5,10 and ∞

$$T(0) = 100\ ^{\circ}F$$

$$T(5) = 134.975\ ^{\circ}F$$

$$T(10) = 155.856\ ^{\circ}F$$

$$T(\infty) = 200\ ^{\circ}F$$

Q – 10.1. A pneumatic PI controller has an output pressure of 10 psi, when the set point and pen point are together. The set point and pen point are suddenly changed by 0.5 in (i.e. a step change in error is introduced) and the following data are obtained.

Time,sec	Psig
0^-	10
0^+	8
20	7
60	5
90	3.5

Determine the actual gain (psig per inch displacement) and the integral time.

Soln:

$e(s) = -0.5/s$

for a PI controller

$Y(s)/e(s) = K_c (1 + \tau_I^{-1}/s)$

$Y(s) = -0.5K_c (1/s + \tau_I^{-1}/s^2)$

$Y(t) = -0.5K_c (1 + \tau_I^{-1} t)$

At $t = 0^+$ y(t) = 8 ➔ Y(t) = 8 – 10 = -2

$2=0.5K_c$

$K_{c\,=}$ 4 psig/in

At t=20 y(t) = 7➔ Y(t) = 7-10 = -3

$3 = 2 (1 + \tau_I^{-1} 20)$

τ_I = 40 sec

Q-10.2. a unit-step change in error is introduced into a PID controller. If $K_C = 10$, $\tau_I = 1$ and $\tau_D = 0.5$. plot the response of the controller P(t)
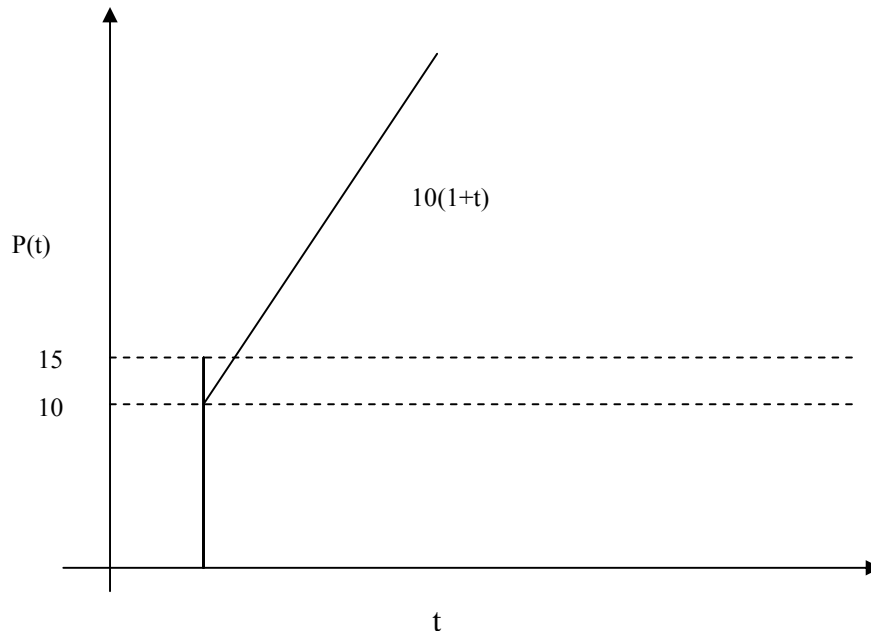
Soln:

$P(s)/e(s) = K_C (1 + \tau_D s+ 1/ \tau_I s)$

For a step change in error

$P(s) = (10/s)(1 + 0.5 s + 1/s)$

$P(s) = 10/s + 5 + 10/s^2$

$P(t) = 10 + 5 \delta(t) + 10 t$

Q – 10.3. An ideal PD controller has the transfer function

$P/e = K_C (\tau_D s + 1)$

An actual PD controller has the transfer function

$P/e = K_C (\tau_D s + 1) / ((\tau_D/\beta) s + 1)$

Where β is a large constant in an industrial controller

If a unit-step change in error is introduced into a controller having the second transfer function, show that

$P(t) = K_C (1 + A \exp(-\beta t/ \tau_D))$

Where A is a function of β which you are to determine. For $\beta = 5$ and $K_C = 0.5$, plot P(t) Vs t/ τ_D. As show that $\beta \rightarrow \infty$, show that the unit step response approaches that for the ideal controller.

Soln:

$P/e = K_C (\tau_D s + 1) / ((\tau_D/\beta) s + 1)$
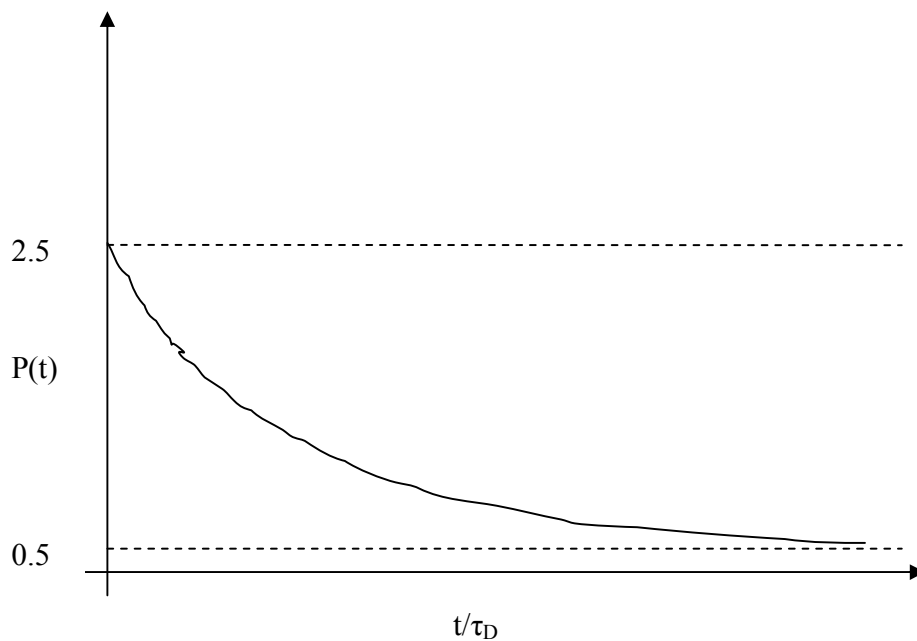
For a step change, $e(s) = 1/s$

$P(s) = K_C s(\tau_D s + 1) / ((\tau_D/\beta) s + 1)$

$$= K_C \left[\frac{1}{s} + \frac{\tau_D \left(1 - \frac{1}{\beta} \right)}{1 + \frac{\tau_D s}{\beta}} \right]$$

$$P(t) \quad = \quad K_C \left[1 + \frac{\tau_D \left(1 - \frac{1}{\beta} \right)}{\frac{\tau_D}{\beta}} e^{-\frac{\beta t}{\tau_D}} \right]$$

$$= \quad K_C \left[1 + (\beta - 1) e^{-\frac{\beta t}{\tau_D}} \right]$$

So, A = β – 1

P(t) = 0.5 (1 + 4 exp(-5t/ τ_D))

t/τ_D

As β → ∞ then τ_D/β → 0 and

P/e = K_C (τ_D s + 1) / ((τ_D/β) s + 1) becomes

P/e = K_C (τ_D s + 1) that of ideal PD controller

Q – 10.4. a PID controller is at steady state with an output pressure of a psig. The set point and pen point are initially together. At time t=0, the set point is moved away from

the pen point at a rate of 0.5 in/min. the motion of the set point is in the direction of lower readings. If the knob settings are

$K_C = 2$ psig/in of pen travel

$\tau_I = 1.25$ min

$\tau_D = 0.4$ min

plot output pressure Vs time

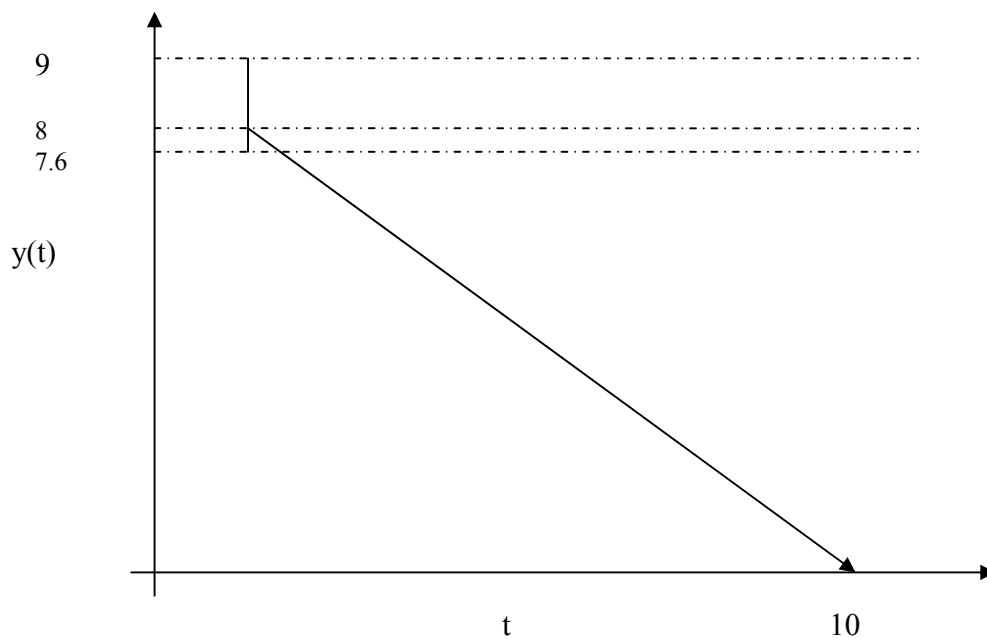**Soln:**

Given $de/dt = -0.5$ in/min

$s\, e(s) = -0.5$

$Y(s)/e(s) = K_C (1 + \tau_D s + 1/ \tau_I s)$

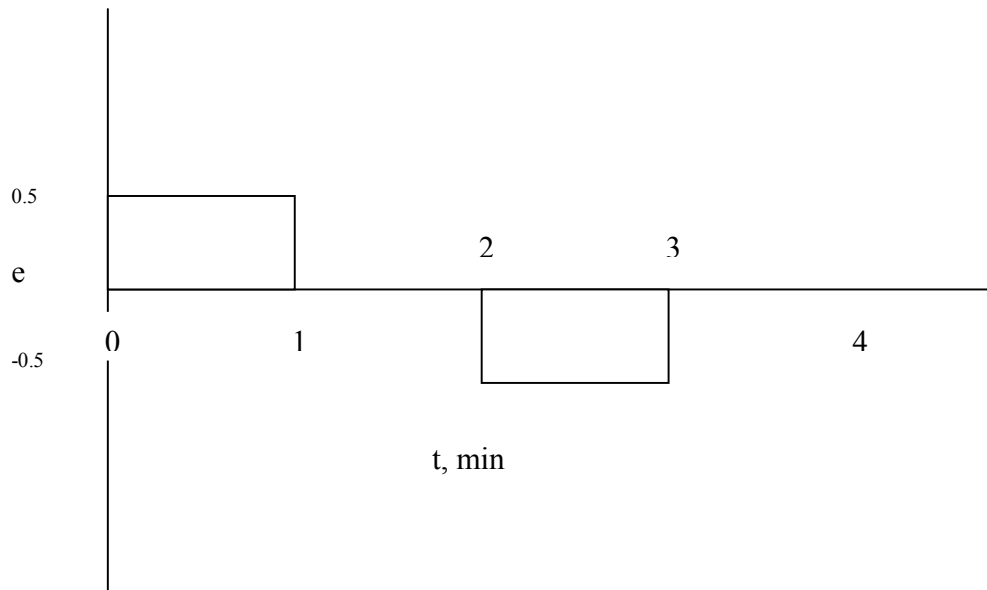$Y(s) = -(1/s + 1/ \tau_I s^2 + \tau_D)$

$Y(t) = -(1 + t/1.25 + 0.4\, \delta(t))$

$Y(t) = y(t) - 9 = - (1 + t/1.25 + 0.4\, \delta(t))$

$y(t) = 8 - 0.8\, t - 0.4\, \delta(t)$

Q – 10.5. The input (e) to a PI controller is shown in the fig. Plot the output of the controller if $K_C = 2$ and $\tau_I = 0.5$ min

0.5

e

-0.5

0   1   2   3   4

t, min

$e(t) = 0.5 (u(t) - u(t-1) - u(t-2) + u(t-3))$

$e(s) = (0.5/s) (1 - e^{-s} - e^{-2s} + e^{-3s})$

$P(s)/e(s) = K_C (1 + (1 / \tau_I s)) = 2 (1 + 2/s)$

$P(s) = (1/s + 2/s^2) (1 - e^{-s} - e^{-2s} + e^{-3s})$

$P(t)$		
$= 1 + 2t$	$0 \le t < 1$
$= 2$	$1 \le t < 2$
$= 5 - 2t$	$2 \le t < 3$
$= 0$	$3 \le t < \infty$

Q – 12.1. Determine the transfer function Y(s)/X(s) for the block diagrams shown. Wxpress the results in terms of Ga, Gb and Gc
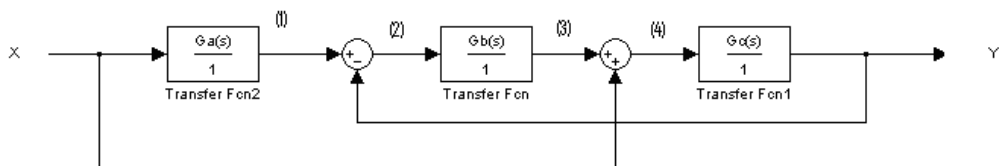
Soln.

(a)   Balances at each node

(1) = GaX

(2) = (1) – Y = GaX – Y

(3) = Gb(2) = Gb(GaX – Y)

(4) = (3) + X = Gb(GaX – Y) + X

Y = Gc(4) = Gc (Gb(GaX – Y) + X)

   = GaGbGcX – GbGcY + GcX

$$\frac{Y}{X} = \frac{Gc(GaGb+1)}{1+GbGc}$$

(b)   Balances at each node

(1) = X – (4)

(2) = Gb(1) = Gb(X – (4))

(5) = GcX/Ga

(3) = Gc(2) = GbGc(X – (4))

(4) = (3) + (5)   --------------------------- 5

   = GbGc(X – (4)) + GcX/Ga

Y = Ga(4)

From the fifth equation

(4) = GbGcX – GbGc(4) + GcX/Ga ----------- 6
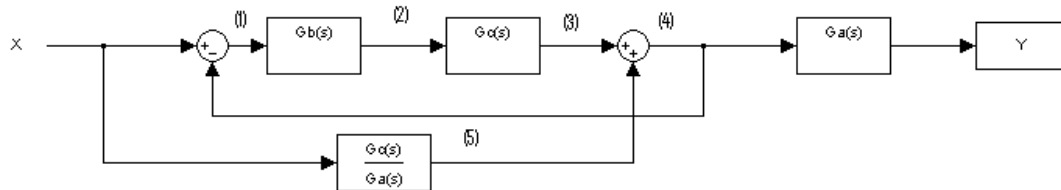
$$(4) = \frac{(GaGbGc + Gc)X}{(1+GbGc)Ga}$$

From the sixth equation

$$\frac{Y}{X} = \frac{(GaGb+1)Gc}{(1+GbGc)}$$

Q – 12.2

Find the transfer function y(s)/X(s) of the system shown

X → (1) → Gb(s) → (2) → Gα(s) → (3) → (4) → Ga(s) → Y

(5) Gα(s) / Ga(s)

Soln:

Balance at each node

    (1) = X – Y   ---------(a)

    (2) = (1) + (3)   ----------(b)

    (3) = G1(2) where G1 = $1/(\tau_1 s + 1)$ ----------(c)

    (4) Y = G2(3) where G2 = $0.5/(\tau_1 s/2 + 1)$ ----------(d)

From (d) and (c)

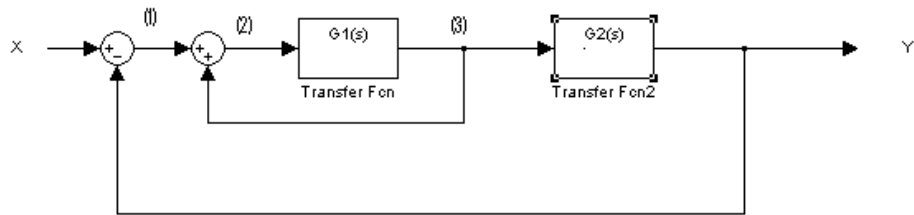   Y = (2)G1G2

      = G1G2 (X – Y + (3))   ----------(e)

Also from (b) and (c)

    (3) = G1((1) + (3))

    (3)$(1 – 1/(\tau_1 s + 1)) = 1/(\tau_1 s + 1)$

    (3) $\tau_1 s = 1$

    (3) = $1/(\tau_1 s) = (X – Y) / (\tau_1 s)$

Substitute this in (e)

$$Y = \frac{0.5}{(\tau_1 s + 1)(\tau_1 s/2 + 1)}\left[1 + \frac{1}{\tau_1 s}\right](X - Y)$$

$$\frac{Y}{X} = \frac{1}{\tau_1^2 s^2 + 2\tau_1 s + 1}$$

Q – 12.3. For the control system shown determine the transfer function C(s)/R(s)

Soln.

Balances at each node

$(1) = R - C$ ------------------(a)

$(2) = 2\,(1) = 2(R - C)$ ----------------(b)

$(3) = (2) - (4) = 2(R - C) - (4)$ ------------------(c)

$(4) = (3)/s = (2(R - C) - (4))/s$ ------------------(d)

$(5) = (4) - C$ ------------------(e)

$C = 2(5)$ ------------------(f)

Solving for (4) using (d)

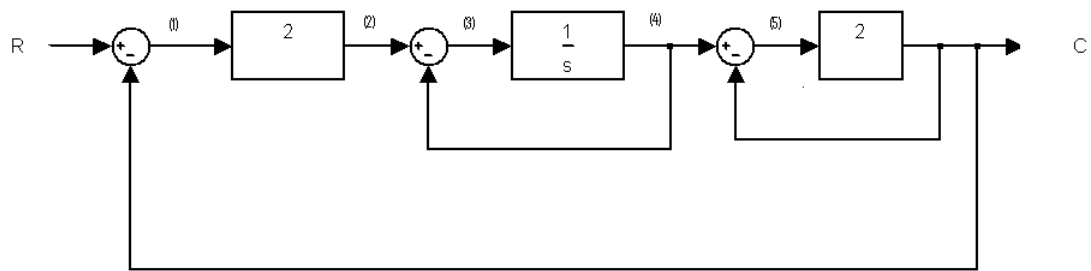$s\,(4) = 2(R - C) - (4)$

$(4) = 2(R - C) / (s + 1)$

Using (e)

$(6) = 2(R - C)/(s + 1) - C$

$$C = 2\left[\frac{2}{s+1}(R - C) - C\right]$$

$4R = C\big((s + 1) + 4 + 2(s + 1)\big)$

$$\frac{C}{R} = \frac{4}{3s + 7}$$

Q – 12.4. Derive the transfer function Y/X for the control system shown

Soln.

Balance at each node

$(1) = (5) + X$ -----------------(a)

$(2) = (1) - (4)$ ----------------(b)

$(3) = (2)/s$ ----------------(c)

$Y = (3)/s$ ----------------(d)

$(5) = 2\,(3)$ ----------------(e)

$(4) = 25Y$ -----------------(f)

From (b)

$(4) = (1) - (2)$

    $= (1) - s\,(3)$ from (c)

    $= (1) - s^2\,Y$ from (d)

    $= (5) + X - s^2\,Y$ from (a)

    $= 2\,(3) + X - s^2\,Y$ from (e)

    $= 2\,s\,Y + X - s^2\,Y$

From (f)

    $Y = (2\,s\,Y + X - s^2\,Y)/25$

    $X = Y(\,25 - 2s + s^2\,)$

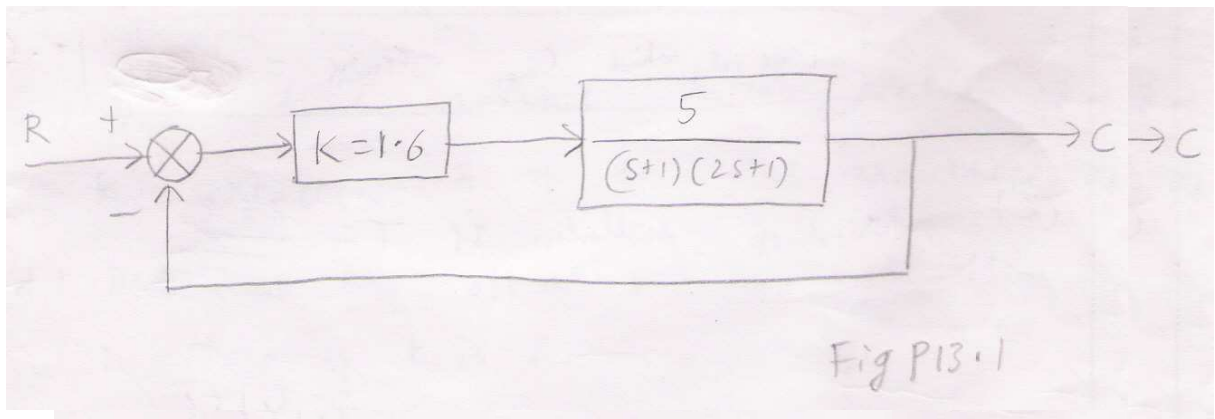 $\dfrac{Y}{X} = \dfrac{1}{s^2 - 2s + 25}$

13.1 The set point of the control system in fig P13.1 given a step change of 0.1 unit. Determine

(a) The maximum value of C and the time at which it occurs.

(b) the offset

(c) the period of oscillation.

Draw a sketch of C(t) as a function of time.

Fig P13.1

$$\frac{C}{R} = \frac{\dfrac{5}{(s+1)(2s+1)}}{1 + K\dfrac{5}{(s+1)(2s+1)}}$$

$$\frac{C}{R} = \frac{8}{2s^2 + 3s + 9}$$

$$R = \frac{0.1}{s}$$

b) $C(\infty) = \underset{S \to 0}{Lt} \dfrac{0.8}{2s^2 + 3s + 9} = \dfrac{0.8}{9} = 0.0889$

offset $= 0.0111$

c) $K = \dfrac{0.8}{9}; \tau = \dfrac{\sqrt{2}}{3}; 2\xi\tau = \dfrac{1}{3} \Rightarrow \xi = \dfrac{1}{2\sqrt{2}}$

$$\text{overshoot} = \exp\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right) = 0.305$$

= Maximum vslue of C = 1.0305*0.0889=0.116

Maximum value of C = 0.116

$$0.116 = \frac{0.8}{9}\left(1 - \frac{1}{\sqrt{1-\xi^2}}\, e^{-\frac{\xi t}{\tau}} \sin\left[\sqrt{1-\xi^2}\,\frac{t}{\tau} + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]\right)$$
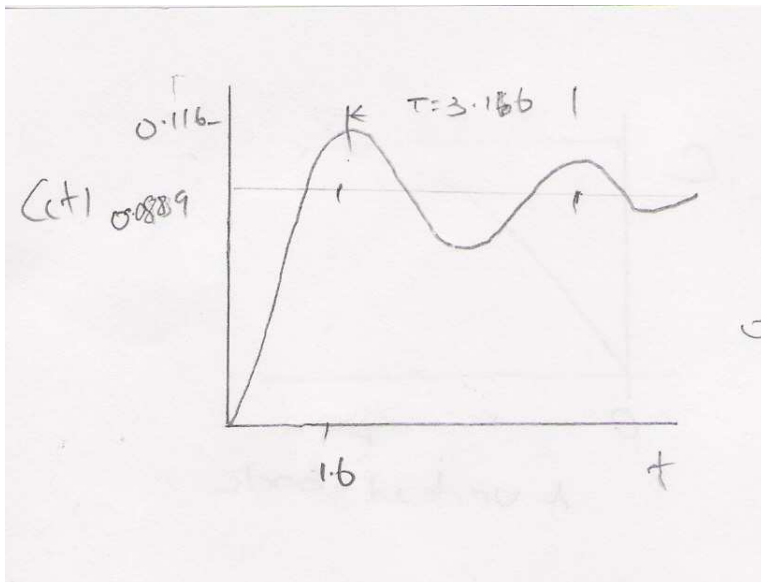
$$t = \frac{\tau}{\sqrt{1-\xi^2}}\tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = 1.6$$

Time at which $C_{\max}$ occurs = 1.6

(c) Period of ociullation is $T = \dfrac{2\pi\tau}{\sqrt{1-\xi^2}} = 3.166$

T =3.166

Decay ratio $= (\text{overshoot})^2 = 0.093$

13.2 The control system shown in fig P 13.2 contains three-mode controller.
(a) For the closed loop, develop formulas for the natural period of oscillation τ and the damping factor ξ in terms of the parameters K, τ_D, τ_I and τ_1.
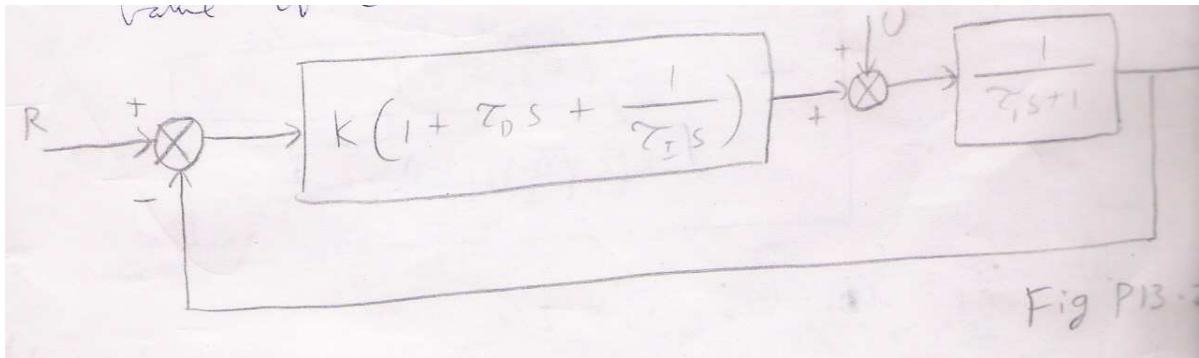(b) Calculate ξ when K is 0.5 and when K is 2.
(c) Do ξ & τ approach limiting values as K increases, and if so, what are these values?
(d) Determine the offset for a unit step change in load if K is 2.
(e) Sktech the response curve (C vs t) for a unit-step change IN LOAD WHEN k is 0.5 and when K is 2.
(f) In both cases of part (e) determine the max value of C and the time at which it occurs.

Fig P13.

$$a)\ \frac{C}{R} = \frac{\dfrac{1}{\tau_1 s + 1} k\left(1 + \tau_D s + \dfrac{1}{\tau_I s}\right)}{1 + \dfrac{k}{\tau_1 s + 1}\left(1 + \tau_D s + \dfrac{1}{\tau_I s}\right)}$$

$$\frac{C}{U} = \frac{\dfrac{1}{(\tau_1 s + 1)}}{1 + \dfrac{1}{\tau_1 s + 1}\left(k\left(1 + \tau_D s + \dfrac{1}{\tau_I s}\right)\right)}$$

$$= \frac{\dfrac{\tau_I s}{k}}{\left(\tau_I \tau_D + \dfrac{\tau_I \tau_I}{k}\right)s^2 + \left(\dfrac{k+1}{k}\right)\tau_I s + 1}$$

$$\frac{C}{R} = \frac{k\left(1 + \tau_D s + \dfrac{1}{\tau_I s}\right)}{\tau_1 s + 1 + k\left(1 + \tau_D s + \dfrac{1}{\tau_I s}\right)}$$

$$\frac{C}{R} = \frac{k\left(\tau_D \tau_I s^2 + \tau_I s + 1\right)}{(k\tau_I \tau_D + \tau_1 \tau_I)s^2 + (k+1)\tau_I s + k}$$

$$\tau^2 = \frac{\tau_I(k\tau_D + \tau_I)}{k}; 2\tau\xi = \frac{(k+1)\tau_I}{k}$$

$$= 2 \times \sqrt{\frac{\tau_I(k\tau_D + \tau_I)}{k}} \quad \xi = \frac{(k+1)\tau_I}{k}$$

$$= \xi = \frac{(k+1)}{2}\sqrt{\frac{\tau_I}{k(k\tau_D + \tau_1)}}$$

$$= T = \frac{2\tau\pi}{\sqrt{1-\xi^2}} = \frac{2\pi \times \sqrt{\dfrac{\tau_I(k\tau_D + \tau_1)}{k}}}{\sqrt{\dfrac{4k(k\tau_D + \tau_1) - (k+1)^2 \tau_D}{2\sqrt{k(k\tau_D + \tau_1)}}}}$$

$$T = \frac{4\pi(k\tau_D + \tau_1)}{\sqrt{4k\left(k\left(\dfrac{\tau_D}{\tau_I}\right) + \left(\dfrac{\tau_1}{\tau_I}\right)\right) - (k+1)^2}}$$

B) $\tau_D = \tau_I = 1; \; \tau_1. = 2$

For $k = 0.5$; $\xi = 0.75\sqrt{\dfrac{1}{0.5(2.5)}} = 0.671$

For $k = 2$; $\xi = 1.5\sqrt{\dfrac{1}{2 \times 3}} = 0.530$

C) $= \xi = \dfrac{1}{2}\sqrt{\dfrac{(k+1)^2 \tau_I}{k(k\tau_D + \tau_1)}} = \dfrac{1}{2}\sqrt{\dfrac{\left(1+\dfrac{1}{k}\right)^2 \tau_I}{\left(\tau_D + \dfrac{\tau_I}{k}\right)}}$

As $k \to \infty, \xi = \dfrac{1}{2}\sqrt{\dfrac{\tau_I}{\tau_1}} = 0.3535$

$$T = \dfrac{4\pi(k\tau_D + \tau_1)}{\sqrt{4k\left(k\left(\dfrac{\tau_D}{\tau_I}\right) + \left(\dfrac{\tau_1}{\tau_I}\right)\right) - (k+1)^2}}$$

$$\tau = \sqrt{\dfrac{k\tau_I\tau_D + \tau_1\tau_I}{k}}$$

$$\tau = \sqrt{\tau_I\tau_D + \dfrac{\tau_I\tau_1}{k}}$$

$$\tau\big|_{K\to\infty} = \sqrt{\tau_I\tau_D} = 1$$

$As\ k \to \infty, T = \dfrac{4\pi\tau_D}{\sqrt{4\dfrac{\tau_D}{\tau_I} - 1}} = 7.2552$

(d) $\dfrac{C}{U} = \dfrac{\dfrac{1}{\tau_1 s + 1}}{1 + \dfrac{k}{\tau_1 s + 1}\left(1 + \tau_D s + \dfrac{1}{\tau_I s}\right)}$

$$\dfrac{C}{U} = \dfrac{\tau_I s}{(k+1)\tau_I s + (\tau_1 + k\tau_D)\tau_I s^2 + k}$$

$$U = \dfrac{1}{s}$$

$$C = \dfrac{\tau_I}{(k+1)\tau_I s + (\tau_1 + k\tau_D)\tau_I s^2 + k}$$

$\underset{S\to 0}{Lt}\ sf(s) = \dfrac{0}{k} = 0$

K=2

For a unit step change in U

$C(\infty) = 0$

Offset = 0

(e) k = 0.5, ξ=**0.671** & τ=**2.236**

$$T = \frac{2\pi\tau}{\sqrt{1-\xi^2}} = 18.95$$

If k = 0.5

$$\frac{C}{U} = \frac{2s}{5s^2 + 3s + 1}; \quad C = \frac{s}{5s^2 + 3s + 1}$$

If k = 2
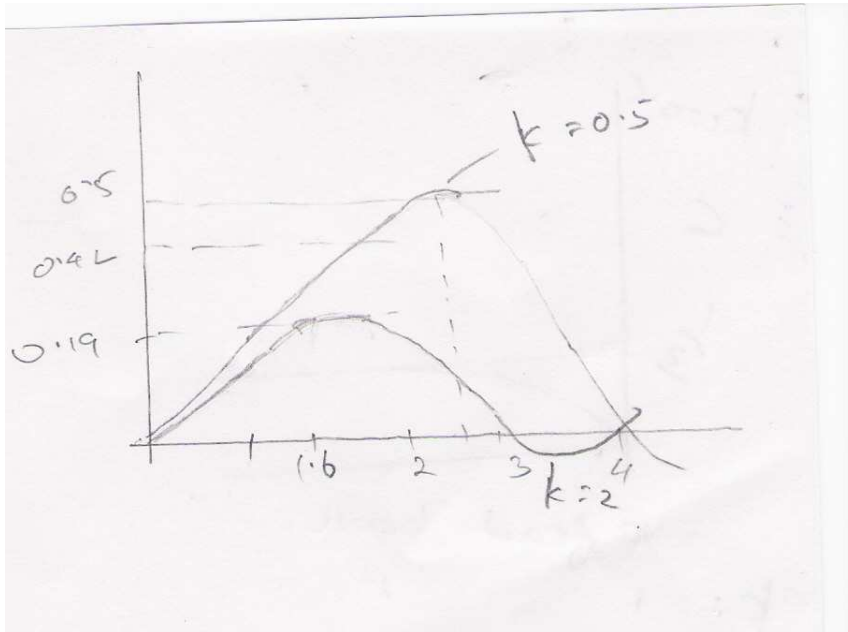
$$\frac{C}{U} = \frac{0.5s}{2s^2 + 1.5s + 1}; C = \frac{0.5}{2s^2 + 1.5s + 1}$$

In general $C(t) = \dfrac{1}{\tau}\dfrac{1}{\sqrt{1-\xi^2}}\, e^{-\frac{\xi t}{\tau}}\, \sin\sqrt{1-\xi^2}\,\dfrac{t}{\tau}$

The maximum occurs at $t = \dfrac{\tau}{\sqrt{1-\xi^2}}\tan^{-1}\dfrac{\sqrt{1-\xi^2}}{\xi}$

If k = 0.5 t_{max} = 2.52 C_{max}=0.42
If k = 2 t_{max} = 1.69 C_{max}=0.19

13.3 The location of a load change in a control loop may affect the system response. In the block diagram shown in fig P 13.3, a unit step chsange in load enters at either location 1 and location 2.

(a) What is the frequency of the transient response when the load enters at location Z?

(b) What is the offset when the load enters at 1 & when it enters at 2?

(c) Sketch the transient response to a step change in U_1 and to a step change in U_2.

Fig P 13.3
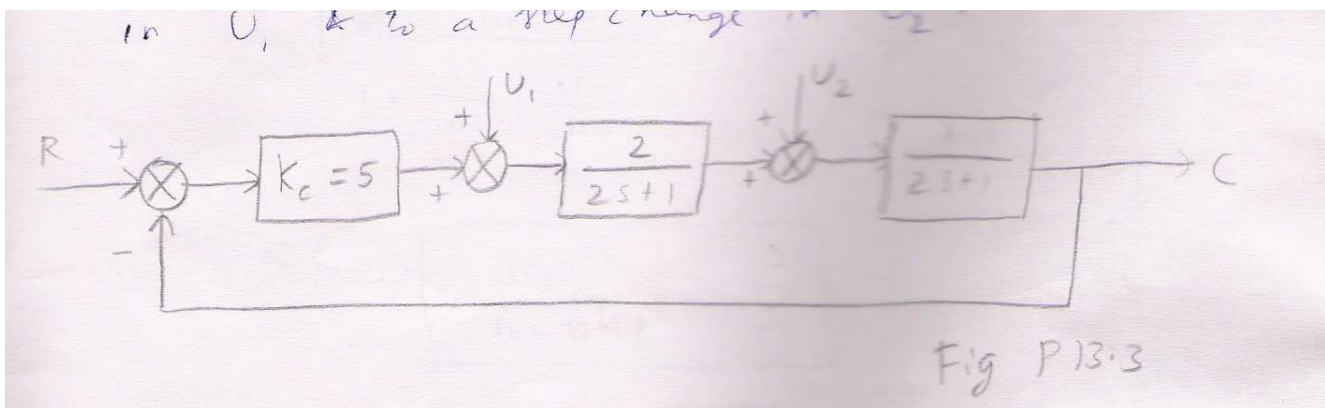
$U_1 = \dfrac{1}{s}; U_2 = 0$

$(S(R-C)+U_1)\left(\dfrac{2}{2s+1}\right)\left(\dfrac{1}{2s+1}\right) = C$

R = 0

$$\frac{C}{U_1} = \frac{\dfrac{2}{(2s+1)^2}}{5 \times \dfrac{2}{(2s+1)^2 + 1}} = \frac{2}{4s^2 + 4s + 11}$$

$$\frac{C}{U_1} = \frac{2}{4s^2 + 4s + 11}$$

$$K = \frac{2}{11}; \tau = \frac{2}{\sqrt{11}}; 2\xi\tau = \frac{4}{11} \Rightarrow \xi = \frac{1}{\sqrt{11}}$$

$$Frequency = f = \frac{1}{T} = \frac{1}{2\pi} \frac{\sqrt{1-\xi^2}}{\tau} = \frac{1}{2\pi} \frac{\sqrt{10}}{2} = 0.2516$$

$C(\infty) = 2/11$

Offset $= 2/11 = 0.182$

$U_1 = 0; U_2 = 1/s$

$$\Rightarrow \left(5 \times \frac{2}{2s+1}(R-C) + U_2 \right) \left(\frac{1}{2s+1} \right) = C$$

R=0

$$\Rightarrow \frac{C}{U_2} = \frac{\dfrac{1}{2s+1}}{\dfrac{10}{2s+1} \times \dfrac{1}{2s+1} + 1}$$

$$\frac{C}{U_2} = \frac{2s+1}{4s^2 + 4s + 11}$$
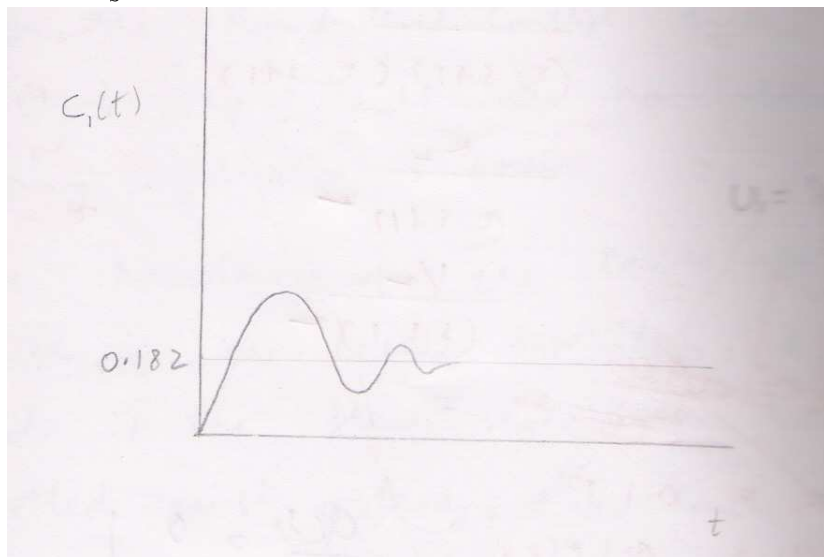
$C(\infty) = 1/11$

Offset $= 1/11 = 0.091$

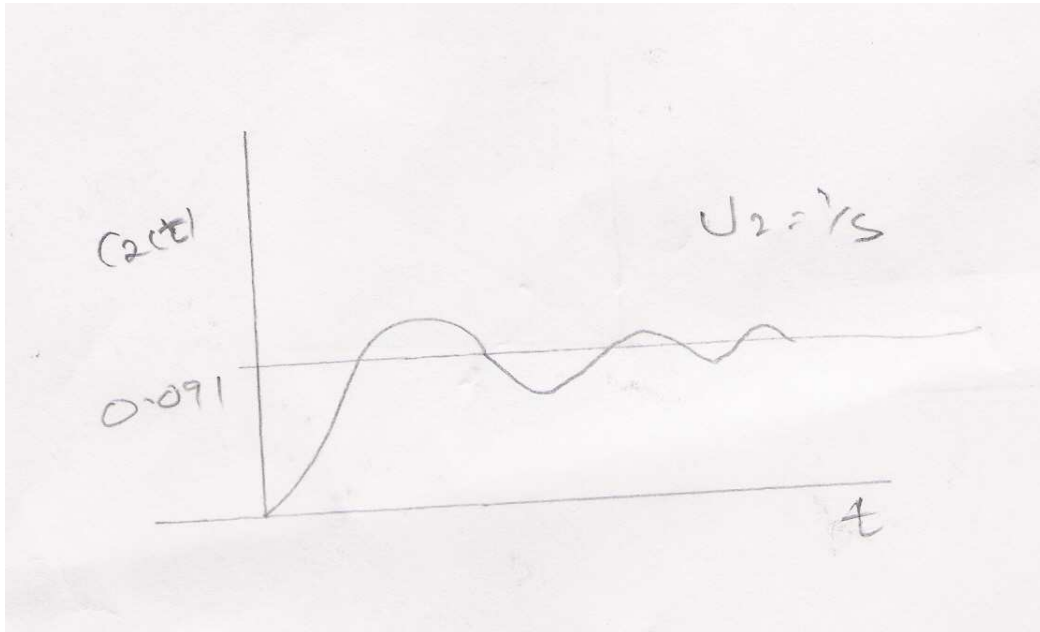   a) if $U_1 = \dfrac{1}{s}$; frequency $= 0.2516$

   if $U_2 = \dfrac{1}{s}$; frequency $= 0.2516$

   b) if $U_1 = \dfrac{1}{s}$; frequency $= 0.182$

if $U_2 = \dfrac{1}{s}$; frequency $= 0.091$

$C_2(t)$

0.091

$U_2 = 1/s$

t

13.5 A PD controller is used in a control system having first order process and a measurement lag as shown in Fig P13.5.

(a) Find the expressions for ξ and τ for the closed –loop response.

(b) If $\tau_1 = 1$ min, $\tau_m = 10$ sec, find K_C so that $\xi = 0.7$ for the two cases: (1) $\tau_D = 0$, (2) $\tau_D = 3$ sec,

(c) Compare the offset and period realized for both cases, and comment on the advantage of adding derivative mode.

$K_c(1+\tau_D s)$ $\dfrac{1}{\tau_1 s+1}$ $\dfrac{1}{\tau_m s+1}$

Fig P13.5
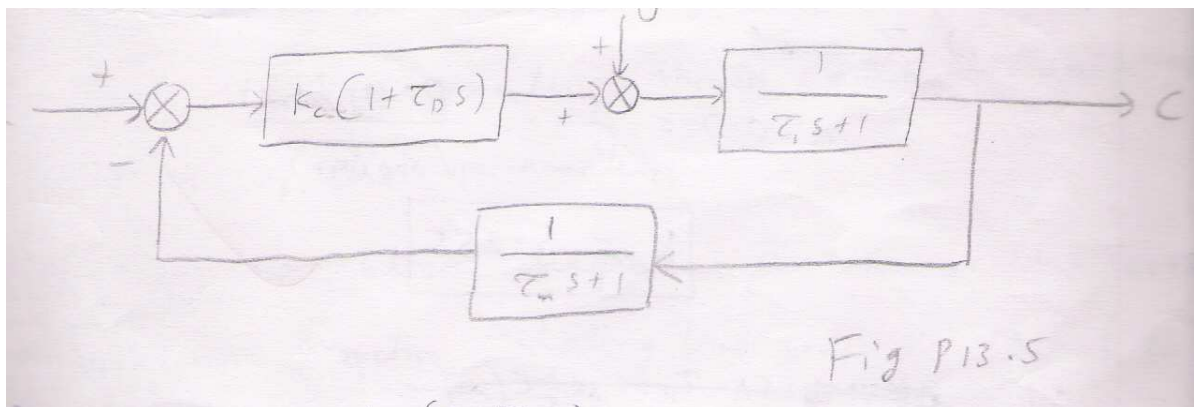
a) $\dfrac{C}{R} = \dfrac{\dfrac{K_C(1+\tau_D s)}{(\tau_1 s+1)}}{1+\dfrac{K_C(1+\tau_D s)}{(\tau_1 s+1)(\tau_m s+1)}}$

$$\frac{C}{R} = \frac{K_C(1+\tau_D s)(1+\tau_m s)}{\tau_1 \tau_m s^2 + (\tau_1 + \tau_m + K_C \tau_D)s + (K_C + 1)}$$

$$= \tau^2 = \frac{\tau_1 \tau_m}{K_C + 1}$$

$$\tau = \sqrt{\frac{\tau_1 \tau_m}{K_C + 1}}$$

$$\xi = \frac{1}{2} \frac{\tau_1 + \tau_m + k_C \tau_D}{\sqrt{\tau_1 \tau_m (k_C + 1)}}$$

b) $\tau_1 = 1 \min; \tau_m = 10s; \xi = 0.7$

$\tau_D = 0$

1) $\Rightarrow 0.7 = \frac{1}{2} \times \frac{70}{\sqrt{600(k_C + 1)}}$

$k_c = 3.167$

$\tau_D = 3s$

2) $\Rightarrow 0.7 = \frac{1}{2} \times \frac{70 + 3k_C}{\sqrt{600(k_C + 1)}}$

$k_C = 5.255$

c)for $R = \frac{1}{s}; c(\infty) = \frac{k_C}{k_C + 1}$

$For\ \tau_D = 0; C(\infty) = 0.76; offset = 0.24$

$for\ \tau_D = 3s; C(\infty) = 0.84; offset = 0.16$

$$for\ \tau_D = 0; period = \frac{2\pi \times \sqrt{\dfrac{600}{4.167}}}{\sqrt{1-\xi^2}} = 105.57 = period$$

$$for\ \tau_D = 3s; period = \frac{2\pi \sqrt{\dfrac{600}{6.255}}}{\sqrt{1-(0.7)^2}} = 86.17s = period$$

Comments:

Advantage of adding derivative mode is lesser offset lesser period

13.6 The thermal system shown in fig P 13.6 is controlled by PD controller.
Data ; w = 250 lb/min; ρ = 62.5 lb/ft^3;
V_1 = 4 ft^3, V_2=5 ft^3; V_3=6ft^3;
C = 1   Btu/(lb)(°F)

Change of 1 psi from the controller changes the flow rate of heat of by 500 Btu/min. the temperature of the inlet stream may vary. There is no lag in the measuring element.
(a) Draw a block diagram of the control system with the appropriate transfer function in each block. Each transfer function should contain a numerical values of the parameters.
(b) From the block diagram, determine the overall transfer function relating the temperature in tank 3 to a change in set point.
(c) Find the offset for a unit steo change in inlet temperature if the controller gain K_C is 3psi/°F of temperature error and the derivative time is 0.5 min.

Fig P13.6
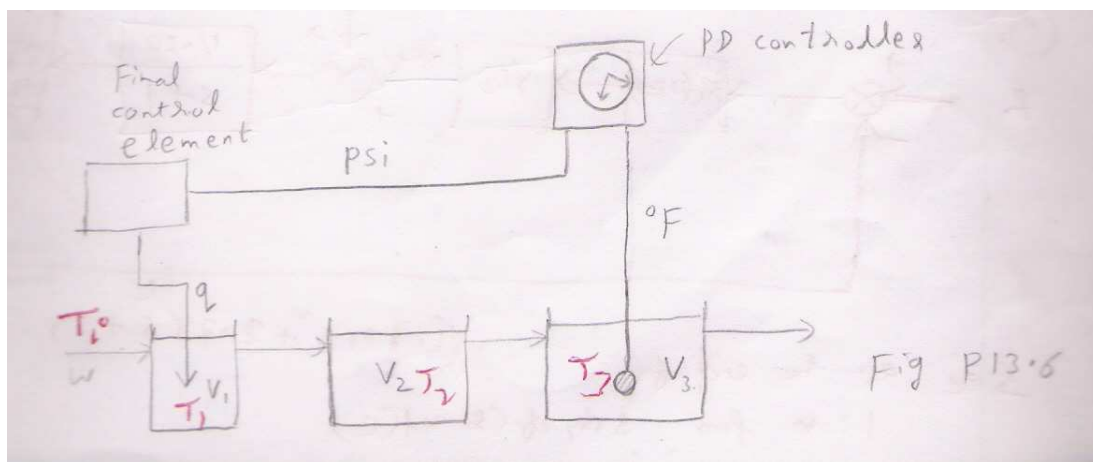
$$WT_0C + q = \rho CV_1(T_1 - T_0) + WT_1C$$

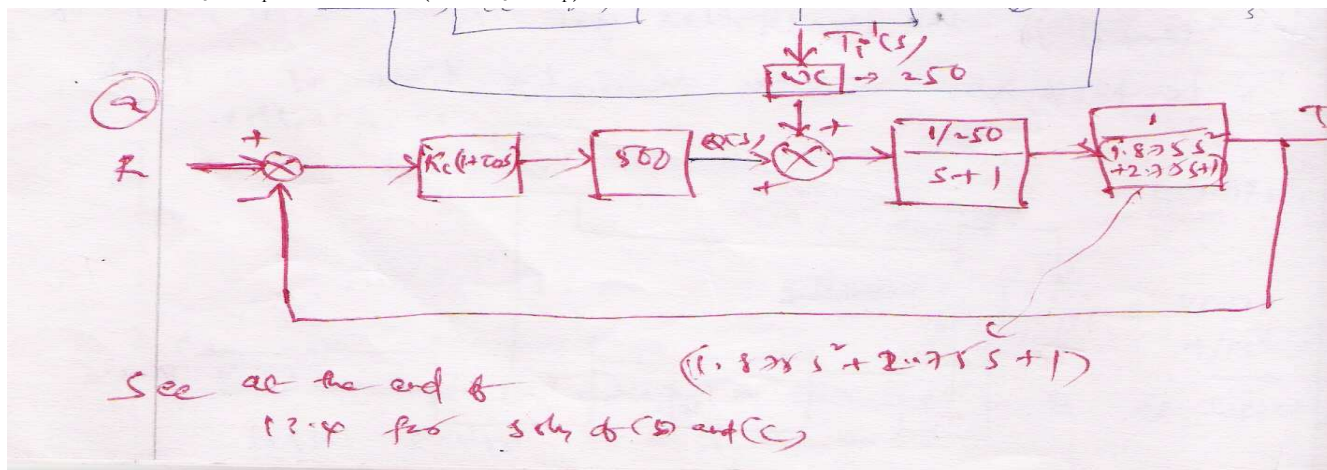$$WT_1C = \rho CV_2(T_2 - T_1) + WT_2C$$

$$WT_2C = \rho CV_3(T_3 - T_2) + WT_3C$$

$$T_0(WC + \rho CV_1) + q = T_1(W\,C + \rho CV_1)$$

$$T_1 = T_0 + \frac{q}{WC + \rho CV_1}$$

$$T_1 = T_2 = T_3$$

$$T_3 = T_0 + \frac{q}{WC + \rho CV_1} \Rightarrow T_3(s) = \frac{q(s)}{(WC + \rho CV_1)s}$$

13.6 (b)

$$\frac{T_3'(s)}{R(s)} = \frac{k_C(1+\tau_D s)\dfrac{2}{(s+1)(1.25s+1)(1.5s+1)}}{1 + k_C(1+\tau_D s)\dfrac{2}{(s+1)(1.25s+1)(1.5s+1)}}$$

$$= \frac{T_3'(s)}{R(s)} = \frac{2k_C(1+\tau_D s)}{(s+1)(1.875s^2 + 2.75s + 1) + 2k_C(1+\tau_D s)}$$

$$\frac{T_3'(s)}{R(s)} = \frac{2k_C(1+\tau_D s)}{1.875s^3 + 4.625s^2 + (3.75 + 2k_C\tau_D)s + 2k_c + 1}$$

c) $k_C=3; \tau_D = 0.5, offset=?, \tau_0'(s) = \dfrac{1}{s}$

$$\frac{T_3'(s)}{T_i'(s)} \underset{s\to 0}{Lt} \frac{1}{1.875s^3 + 4.625s^2 + (3.75 + 2k_C\tau_D)s + 2k_C + 1}$$

$$= \frac{1}{2k_C+1} = \frac{1}{7} = 0.143$$

Offset $=0.143$

13.7 (a) For the control system shown in fig P 13.7, obtain the closed loop transfer function C/U.

   (b) Find the value of K_C for whgich thre closed loop response has a ξ of 2.3.

   (c) find the offset for a unit-step change in U if $K_C = 4$.

Fig P 13·7
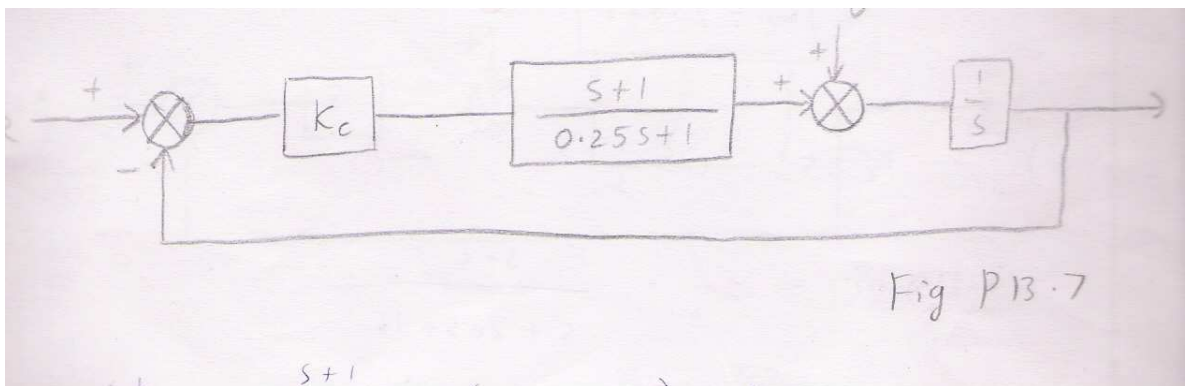
$$\left(K_C \times \frac{s+1}{0.25s+1}(R-C)+U \right)\frac{1}{s} = C$$

$$= \frac{C}{U} = \frac{\dfrac{1}{s}}{1+\dfrac{K_C}{s}\cdot\dfrac{s+1}{0.25s+1}}$$

$$\frac{C}{U} = \frac{0.25s + 1}{0.25s^2 + s + K_C(s+1)}$$

$$\frac{C}{U} = \frac{s + 4}{s^2 + 4(K_C + 1)s + 4K_C}$$

   b) ξ=2.3

$$\tau = \frac{1}{\sqrt{4K_C}}; 2\xi\tau = \frac{K_C + 1}{K_C}$$

$$= \frac{1}{\sqrt{4K_C}} 2 \times 2.3 = \frac{K_C + 1}{K_C}$$

$$= \frac{K_C + 1}{\sqrt{K_C}} = 2.3$$

K$_C$=2.952

   C) K$_C$=4,U = 1/s

$$= C = \frac{1}{s} \times \frac{s + 4}{s^2 + 20s + 16}$$

$$C(\infty) = \frac{1}{4}$$

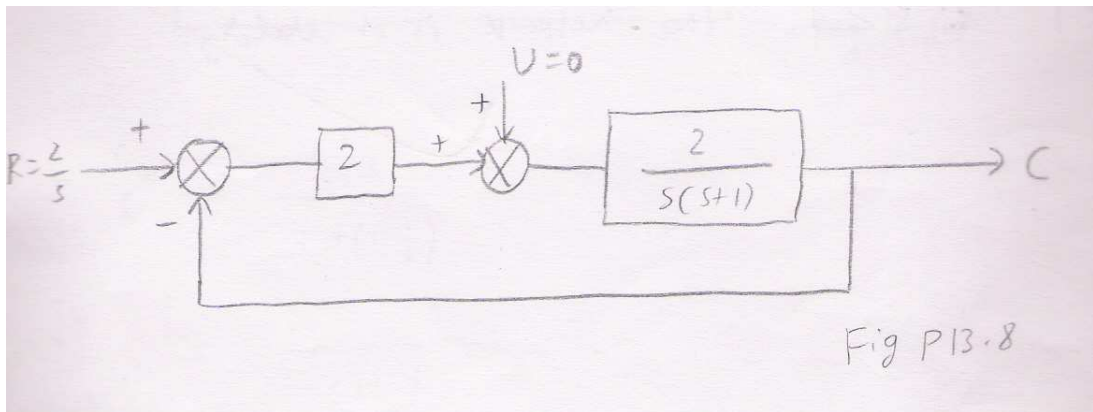offset = 0.25.

13.8 For control system shown in Fig 13.8
    (a) C(s)/R(s)
    (b) C(∞)
    (c) Offset
    (d) C(0.5)
    (e) Whether the closed loop response is oscillatory.

Fig P13.8

(a) $\dfrac{C}{R} = \dfrac{\dfrac{4}{s(s+1)}}{1 + \dfrac{4}{s(s+1)}}$

$\dfrac{C}{R} = \dfrac{4}{s^2 + s + 4}$

b) $C(\infty) = 2*1 = 2$

$C(\infty) = 2$

C) offset $= 0$

d) $\tau = \dfrac{1}{2}; 2\xi\tau = \dfrac{1}{4} \Rightarrow \xi = \dfrac{1}{4}$

$\dfrac{C(t)}{2} = 1 - \dfrac{1}{\sqrt{1 - \left(\dfrac{1}{4}\right)^2}} e^{-\frac{t}{2}} \sin\left[\dfrac{\sqrt{15}}{4}\dfrac{t}{\tau} + \tan^{-1}\sqrt{15}\right]$
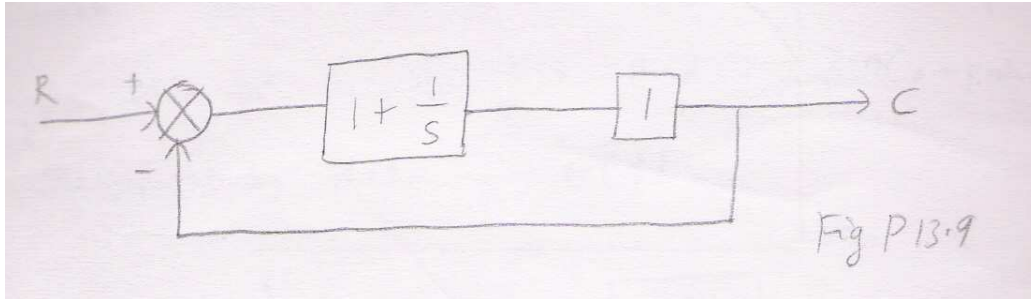
$= C(0.5) = 2\left[1 - \dfrac{4}{\sqrt{15}} e^{-\frac{1}{4}} \sin\left[\dfrac{\sqrt{15}}{4} + \tan^{-1}\sqrt{15}\right]\right]$

$C(0.5) = 0.786$

.e) $\xi < 1$, the response is oscillatory.

13.9 For the control system shown in fig P13.9, determine an expression for C(t)

if a unit step change occurs in R. Sketch the response C(t) and compute C(2).

Fig P13.9

$$\frac{C}{R} = \frac{1 + \dfrac{1}{s}}{1 + \left(1 + \dfrac{1}{s}\right)}$$

$$\frac{C}{R} = \frac{s + 1}{2s + 1}$$

$$R = \frac{1}{s}$$

$$C = \frac{s + 1}{s(2s + 1)} = \frac{1}{s} + \frac{-1}{2s + 1}$$

$$C(t) = 1 - \frac{1}{2}e^{-\frac{t}{2}}$$

$$C(2) = 0.816$$

13.10 Compare the responses to a unit-step change in a set point for the system shown in fig P13.10 for both negative feedback and positive feedback. Do this for K_C of 0.5 and 1.0. compare the responses by sketching C(t).

Fig P13.10
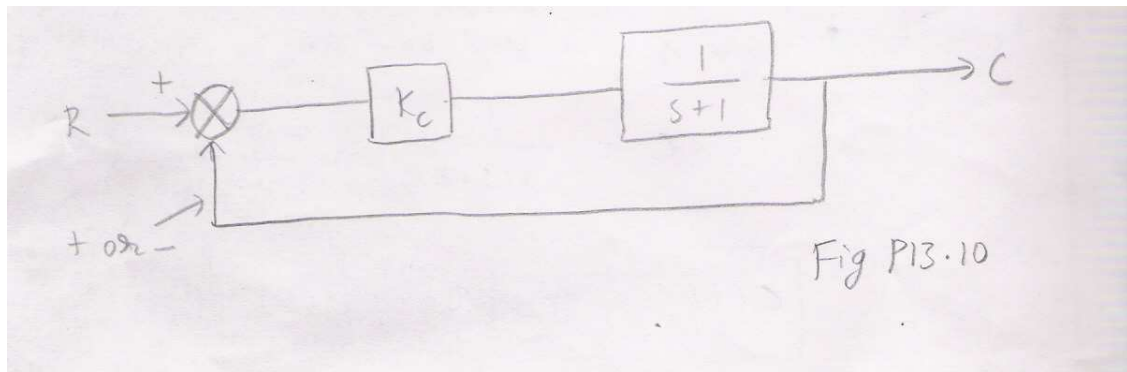
-ve feed back :

$$C = \frac{K_C}{s(s + (K_C + 1))}$$

+ve feed back

$$\frac{C}{R} = \frac{K_C \times \dfrac{1}{s+1}}{1 - K_C \dfrac{1}{s+1}}$$

$$C = \frac{K_C}{s(s + (1 - K_C))}$$

For $K_C = 0.5$, response of -ve feed back is

$$C = \frac{1}{s(2s + 3)} = \frac{\dfrac{1}{3}}{s} + \frac{-\dfrac{2}{3}}{2s + 3}$$

$$C(t) = \frac{1}{3} - \frac{1}{3}e^{-\frac{3t}{2}} = \frac{1}{3}\left(1 - e^{-\frac{3t}{2}}\right)$$

response of +v feed back is

$$C = \frac{1}{s(2s+1)} = \frac{1}{s} + \frac{-2}{2s+1}$$

$$C(t) = 1 - e^{-\frac{t}{2}}$$

For $K_C = 1$, response of -ve feed back is

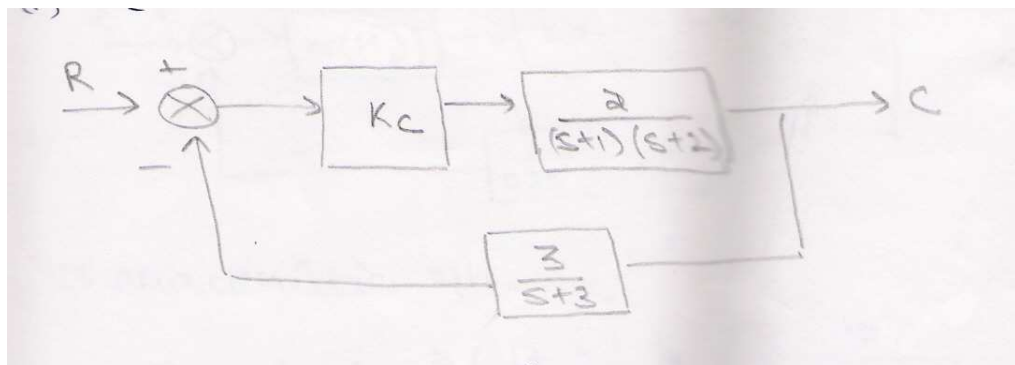$$C = \frac{1}{s(s+2)} = \frac{\frac{1}{2}}{s} + \frac{-\frac{1}{2}}{s+2}$$

$$C(t) = \frac{1}{2} - \frac{1}{2}e^{-2t}$$

response of +ve feed back is

$$C = \frac{1}{s^2}$$

$$C(t) = t$$

14.1 Write the characteristics equation and construct Routh array for the control system shown . it is stable   for   (1) Kc= 9.5,(ii) KC =11; (iii) Kc= 12

Characteristics equation

$$1 + \frac{6Kc}{(s+1)(s+2)(s+3)} = 0$$

$$or \ (s+1)(s+2)(s+3) + 6Kc = 0$$

$$(s^2 + 6s + 11s + (6 + 6Kc) = 0$$

$$s^3 + 6s^2 + 11s + (6 + 6Kc) = 0$$

Routh array

s^3	1	11
s^2	6	$(6 + 6Kc)$
s	$6(1 + Kc)$	

For Kc=9.5

= 10-(Kc)= 10-9.5=0.5>0 therefore stable.

For Kc=11

= 10-(Kc)= 10-11=-1<0 therefore unstable

For Kc=121

= 10-(Kc)= 10-12=--2<0 therefore unstable

14.2 By means of the routh test, determine the stability of the system shown when $K_C = 2$.

Characteristic equation

$$1 + 2\left(1 + \frac{3}{s}\right)2\left(\frac{10}{2s^2 + 4s + 10}\right) = 0$$

$$(2s^2 + 4s + 10)s + 2(s + 3).2.10 = 0$$

$$2s^3 + 4s^2 + 10s + 40s + 120 = 0$$

$$2s^3 + 4s^2 + 50s + 120 = 0$$

$$s^3 + 2s^2 + 25s + 60 = 0$$

Routh Array

```
1      25
2      60
-10/2
```

The system is unstable at Kc = 2.

14.4 Prove that if one or more of the co-efficient $(a_0, a_1, \ldots a_n)$ of the characteristic equation are negative or zero, then there is necessarily an unstable root

Characteristic equation :

$$a_0 x^n + a_1 x^{n-1} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_n = 0$$

$$a_0 (x^n + a_1 / a_0 x^{n-1} + \ldots\ldots\ldots\ldots a_n / a_0) = 0$$

$$a_0 (x - \alpha_1)(x - \alpha_2)\ldots\ldots\ldots\ldots(x - \alpha_n) = 0$$

We have $\alpha_1, \alpha_2 \ldots\ldots\ldots\ldots\ldots \alpha_n < 0$

As we know the second co-efficient a_1/a_0 is sum of all the roots

$$\frac{a_1}{a_0} = (-1)^2 \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j \right] / 2$$

Therefore sum of all possible products of two roots will happen twice as $\alpha_1 \alpha_2$ dividing the total by 2.

$$\alpha_i \alpha_j > 0 \ (\alpha_i < 0 \ \ \alpha_j < 0)$$

And

$$\therefore a_2 / a_0 > 0 \Rightarrow a_2 > 0$$

Similarly

$$\frac{a_j}{a_0} = (-1)^j (\text{ sum of aoll possible products of } j \text{ roots})$$

if $j = even$ $(-1)^j$ *is* 1 *and the sum is* > 0 *so* $a_j / a_0 > 0$

if $j = odd$ $(-1)^j$ *is* (-1) *and the sum is* < 0 *so* $\frac{a_j}{a_0}$ *is again* > 0

in both case $a_j / a_0 > 0$

so $a_j > 0 (for\ j = 1, ... n)$

14.5 Prove that the converse statement of the problem 14.4 that an unstable root implies that one or more co-efficient will be negative or zero is untrue for all co-efficient ,n>2.

Let the converse be true, always .Never if we give a counter example we can contradict.
Routh array

$$s^3 + s^2 + 2s + 3$$

s^3 1   2
s^2 1   3
s -1 0
s^0

System is unstable even when all the coefficient are greater than 0; hence a contradiction,

14.6 Deduce an expression for Routh criterion that will detect the Presence of roots with real parts greater than σ for any rectified $\sigma > 0$

Characteristic equation

$$a_0 x^n + a_1 x^{n-1} + ... + a_n = 0$$

Routh criteria determines if for any root, real part > 0.

Now if we replace x by X such that

$.x + \sigma = \mathbf{X}$

Characteristic equation becomes

$$a_0 (X - \sigma)^n + a_1 (X - \sigma)^{n-1} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_n = 0$$

Hence if we apply Routh criteria,

We will actually be looking for roots with real part $> \sigma$ **rather than >0**

$$a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_n = 0$$

Routh criterion detects if any root α_j is greater than zero.

Is there any $x = \alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_j, \ldots\ldots\alpha_n > 0 ----- (1)$

Now we want to detect any root

$\alpha\, j > -\sigma$

$\alpha\, j > 0$

$From(1)$

$$x = \alpha_1, \alpha_2, \ldots\ldots\ldots\ldots\ldots\ldots\alpha_j, \ldots\ldots\ldots\ldots\ldots\ldots\alpha_n, > 0$$

implies is there any

$$x = \alpha_1 > 0$$
$$x = \alpha_2 > 0$$
.

.

.

$$x = \alpha_j > 0$$
.

.

.

$$x = \alpha_n > 0$$
add σ on both sides

is there any

$$x + \sigma = \alpha_1 + \sigma > 0$$
$$x + \sigma = \alpha_2 + \sigma > 0$$
.

.

.

$$.x + \sigma = \alpha_j + \sigma > 0$$
.

.

.

$$x + \sigma = \alpha_n + \sigma > 0$$

so, Let $X = x + \sigma$

and apply Routh criteria to $\det ect$ any $\alpha_j + \sigma > 0$

or $\alpha_j > -\sigma$

14.7 Show that any complex no S₁ satisfying $|S| < 1$, yields a value of

$$Z = \frac{1+s}{1-s} \text{ that satisfies Re(Z)>0,}$$

Let S=x+iy, $\sqrt{x^2 + y^2} < 1$

$$Z = \frac{1+s}{1-s}$$

$$\frac{(1+x)+iy}{(1-x)-iy}\cdot\frac{(1-x)+iy}{(1-x)+iy}$$

$$\frac{(1-x^2+(1=x+1-x)iy-y^2)}{1+x^2-2x+y^2}$$

$$=\frac{1-(x^2+y^2)+2iy}{1-2x+(x^2+y^2)}$$

$$\text{Re}(Z)=\frac{1-(x^2+y^2)}{1-2x+(x^2+y^2)}$$

$$if\ \ \text{Re}(z)>0\ \ then\ 1-(x^2+y^2)>0\ \ and$$

$$1-2x+(x^2+y^2)>0$$

$$we\ have\ \sqrt{x^2+y^2}\ <1$$
$$x^2+y^2<1$$

$$Ranges\ are\ \ -1<x<1$$
$$-1<y<1\qquad Points\ \ in\ \ the\ unit\ circle$$

$$1-(x^2+y^2)>0\ \ is\ \ true\ \ therefore\ \ x^2+y^2<1$$

$$Now$$
$$1+(x^2+y^2)-2x$$

$$if\ \ x=-1\ \&\ \ y=0\ \ then\ it\ is\ 4$$
$$if\ \ x=1\ \&\ \ y=0\ then\ it\ is\ 0$$

$$0<(1+(x^2+y^2)-2x)<4$$

$$\text{Re}(z)>0$$

$$example:$$
$$if\ \ s=(0.5+i0.5)\ the\ system\ is\ unstable\ due\ to\ the\ real\ part$$

$$L^{-1}\left[\frac{1}{s-(0.5+i0.5)}\right]$$

$$L^{-1}\left[\frac{1}{s-(0.5+i0.5)}\right]=e^{0.5t}(Cos\ 0.5t+Sin\ 0.5t)$$

14.8 For the output C to be stable, we analyze the characteristic equation of the system

$$1+\frac{1}{\tau_I s}\frac{1}{(\tau_1 s+1)(\tau_2 s+1)}\times(\tau_3 s+1)=0$$

$$\tau_I s(\tau_1 s^2\tau_2+\tau_1 s+\tau_2 s+1)+\tau_3 s+1=0$$

$$\tau_I\tau_1\tau_2 s^3+\tau_I(\tau_1+\tau_2)s^2+(\tau_I+\tau_3)s+1=0$$

Routh Array

s^3	$\tau_I\tau_1\tau_2$	$\tau_I+\tau_3$
s^2	$\tau_I(\tau_1+\tau_2)$	1
s	α	0
s^0	1	

$$\alpha=\frac{\tau_I(\tau_1+\tau_2)(\tau_I+\tau_3)-\tau_I\tau_1\tau_2}{\tau_I(\tau_1+\tau_2)}$$

Now

(1) $\tau_I\tau_2\tau_1>0$

Since $\tau_1\ \&\ \tau_2$ are process time constant they are definitely +ve
$\tau_1>0;\tau_2>0$

(2) $\tau_I(\tau_1+\tau_2)>0$

(3) $\alpha>0\Rightarrow\tau_I(\tau_1+\tau_2)(\tau_I+\tau_3)>\tau_I\tau_1\tau_2$
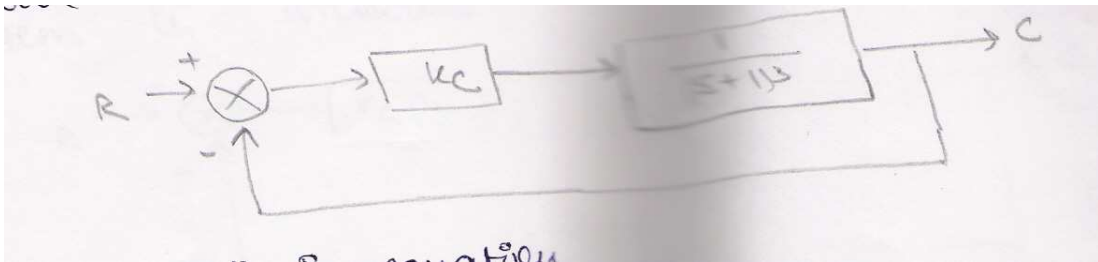
$$\tau_1\tau_I + \tau_1\tau_3 + \tau_2\tau_I + \tau_2\tau_3 - \tau_1\tau_2 > 0$$

$$\tau_I(\tau_1 + \tau_2) > \tau_1\tau_2 - \tau_3(\tau_1 + \tau_2)$$

$$\tau_I > \frac{\tau_1\tau_2}{\tau_1 + \tau_2} - \tau_3$$

also $\tau_I > 0$

14.9 In the control system shown in fig find the value of Kc for which the system is on the verge of the instability. The controller is replaced by a PD controller, for which the transfer function is Kc(1+s). if Kc = 10, determine the range for which the system is stable.

Characteristics equation

$$1 + \frac{6Kc}{(s+1)(s+2)(s+3)} = 0$$

$or \ (s+1)(s+2)(s+3) + 6Kc = 0$

$(s^2 + 3s + 2)(s+3) + 6Kc = 0$

$s^3 + 6s^2 + 11s + (6 + 6Kc) = 0$

Routh array

$s^3 \quad 1 \quad 3$

$s^2 \quad 3 \quad 1 + Kc$

$s \quad \quad 3 - \left(\frac{1 + Kc}{3}\right)$

For verge of instability $\quad 3 = \left(\dfrac{1 + Kc}{3} \right)$

$$Kc = 8$$

Characteristics equation

$$1 + \dfrac{10(1 + kcs)}{(s + 1)3} = 0$$

$$s^3 + 3s^2 + s(3 + 10Kcs) + 11 = 0$$

Routh Array

$s^3 \quad 1 \qquad 3 + 10\tau_D$
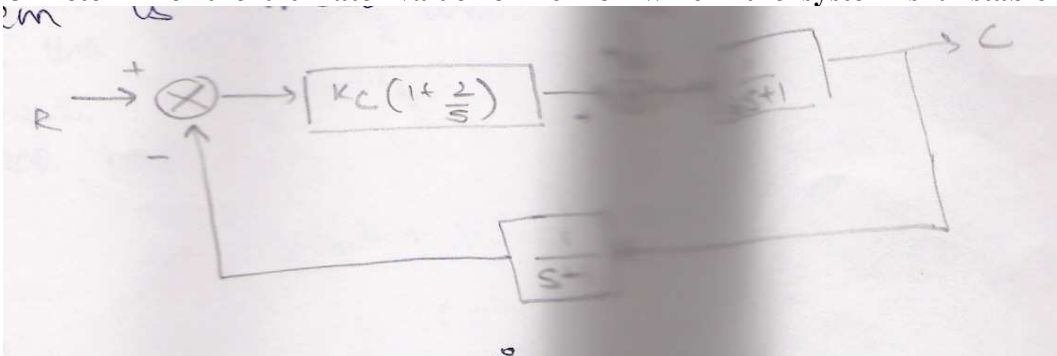
$s^2 \quad 3 \qquad 11$

s

$3(3 + 10\tau_s) > 11 \ \ for \ vege$

$30\tau_S > 2$

$\tau_D > 2/30$

14.10 (a) Write the characteristics equation for the central system shown (b) Use the routh criteria to determine if the system is stable for Kc=4 © Determine the ultimate value of Kc for which the system is unstable

(a) characteristics equation

$$1 + kc\left(\frac{s+2}{3}\right)\left(\frac{1}{2s=1}\right)\frac{1}{(s+1)} = 0$$

$$(s^2 + s)(2s + 1) + kc(s + 2) = 0$$

$$2s^3 + 3s^2 + (1 + kc)s + 2kc = 0$$

$$s^3 + 3s^2 + 3s + (1 + kc) = 0$$

Kc=4Routh array

$$s^3 \quad 2 \quad 5$$
$$s^2 \quad 3 \quad 8$$
$$s \quad -1/3$$

not stable

$$\frac{3(1 + kc) - 4kc}{3} = 0$$
$$3 - Kc = 0; Kc = 3$$

For verge of instability

14.11 for the control shown, the characteristics equation is
$$s^4 + 4s^3 + 6s^2 + 4s + (1 + k) = 0$$
(a) determine value of k above which the system is unstable.
(b) Determine the value of k for which the two of the roots are on the imaginary axis, and determine the values of these imaginary roots and remaining roots are real.

$$s^4 + 4s^3 + 6s^2 + 4s + (1 + k) = 0$$

$$s^4 \quad 1 \quad 6(1+k)$$
$$s^3 \quad 4 \quad 4$$
$$s^2 \quad 5 \quad 1+k$$
$$s \quad 4-\frac{4}{5}(1+k)$$
$$1 \quad 1+k$$

For the system to be unstable

$$4\left(1-\left(\frac{1+k}{5}\right)\right)<0$$

$$1<\frac{1+k}{5}$$

$$k>4$$
$$1+k<0$$
$$k<-1$$
$$k>-1$$

The system is stable at -1<k<4

(b) For two imaginary roots

$$4=\frac{4}{5}(1+k); k=4$$

Value of complex roots
$$5s^2+5=0$$
$$s=\pm i$$

$$s^2+1 \quad s^4+4s^3+6s^2+4s+5 \quad s^2+4s+5$$

$$s^4 + 0 + s^2$$

$$4s^3 + 5s^2$$

$$4s^3 + 0 + 4s$$

$$5s^2 + 5$$

$$5s^2 + 5$$

$$0$$

SOLUTION:

$$s = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$$

PART 2

LIST OF USEFUL BOOKS FOR PROCESS CONTROL

1.   **PROCESS CONTROL BY R.P VYAS, CENTRAL TECHNO PUBLICATIONS, INDIA (** *WIDE VARIETY OF SOLVED PROBLEMS ARE AVAILABLE IN THIS BOOK)*

2. **ADVANCED CONTROL ENGINEERING BY RONALD.S .BURNS , BUTTERWORTH AND HIENEMANN.**

3. **PROCESS MODELLING SIMULATION AND CONTROL FOR CHEMICAL ENGINEERS, WILLIAM.L.LUYBEN, MCGRAW HILL.**

4.   **A MATHEMATICAL INTRODUCTION TO CONTROL THEORY BY SCHOLOMO ENGELBERG, IMPERIAL COLLEGE PRESS**

LIST OF USEFUL WEBSITES

**www.msubbu.com FOR BLOCK DIAGRAM REDUCTION AND
OTHER CHEMICAL ENGG. LEARNING RESOURCES**

**Readings,Recitations,Assignments,Exams,StudyMaterials,Discussion Group,Video Lectures
now study whatever u want with respect to chemical
engg.**

http://ocw.mit.edu/OcwWeb/index.htm

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